Convex Preferences

Take one particular consumption bundle $\mathbf{x}$: the **at least as good as set** $\succeq(\mathbf{x})$ is the set of all consumption bundles which the person finds at least as good as the "reference bundle" $\mathbf{x}$.

In other words, $\succeq(\mathbf{x})$ is the set consisting of all the consumption bundles on the indifference curves through $\mathbf{x}$, or on better indifference curves. (If preferences are strictly monotonic, then $\succeq(\mathbf{x})$ consists of all the bundles on or above the indifference curve through $\mathbf{x}$.)

Preferences are said to be convex if, for **any** reference bundle $\mathbf{x} \in X$, the at least as good as set $\succeq(\mathbf{x})$ is a **convex set**.

An (equivalent) alternate definition : preferences are convex if : for any consumption bundle $\mathbf{x}$, if $\mathbf{x}^1 \succeq \mathbf{x}$, and if $\mathbf{x}^2 \succeq \mathbf{x}$, and if

$$\mathbf{x}^3 \equiv t\mathbf{x}^1 + (1-t)\mathbf{x}^2$$

where t is any number between 0 and 1,

then

$\mathbf{x}^3 \succeq \mathbf{x}$.

That is, if $\mathbf{x}^1$ and $\mathbf{x}^2$ are both at least as good as $\mathbf{x}$, then any **convex combination** of $\mathbf{x}^1$ and $\mathbf{x}^2$ is at least as good as $\mathbf{x}$. A convex combination of two vectors is defined as any point between the two vectors, on the line connecting the two vectors : every point on the line connecting $\mathbf{x}^1$ and $\mathbf{x}^2$ can be expressed as some fraction of the way along that line, $t\mathbf{x}^1 + (1-t)\mathbf{x}^2$, where t is the fraction of the way along the line that the point is.

Strictly Convex Preferences

Strict convexity of preferences is a **stronger** property than just plain convexity. Preferences are strictly convex if : for any consumption bundle $\mathbf{x}$, if $\mathbf{x}^1 \succeq \mathbf{x}$, and if $\mathbf{x}^2 \succeq \mathbf{x}$, (with $\mathbf{x}^1 \neq \mathbf{x}^2$) then for any $0 < t < 1$,

$$t\mathbf{x}^1 + (1-t)\mathbf{x}^{\mathbf{2}} \succ \mathbf{x}$$

So, in two dimensions, with strictly monotonic preferences, strict convexity says that if two consumption bundles are each on the same indifference curve as $\mathbf{x}$, then any point on a line connecting these two points (except for the points themselves) will be on a **higher** indifference curve than $\mathbf{x}$.

In two dimensions, if indifference curves are straight lines, then preferences are convex, but **not** strictly convex.

Utility Functions

A utility function is **quasi–concave** if and only if the preferences represented by that utility function are **convex**.

A utility function is **strictly quasi–concave** if and only if the preferences represented by that utility function are **strictly convex**.