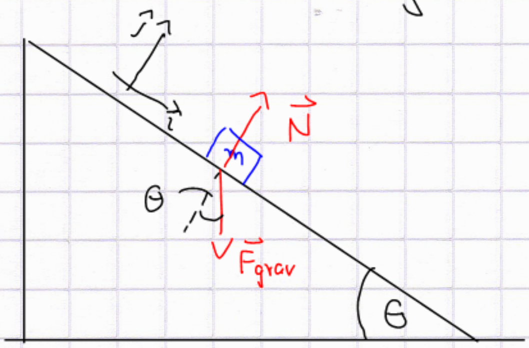
Inclined planes and another 2D example

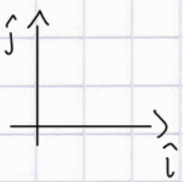
(i) w/o friction



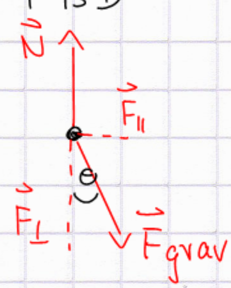
Newton-2:

$$m\vec{a} = \vec{F}_{net} = \vec{F}_{grav} + \vec{N}$$

Use rotated coordinate system to decompose into components:



FBD



$$\vec{F}_{grav} = F_{\parallel}\hat{i} + F_{\perp}\hat{j}$$

$$F_{\parallel} = mg\sin\theta = F_{net}$$

$$F_{\perp} = -mg\cos\theta$$

$\hat{i}$: $\quad ma_x = F_{net} = mg\sin\theta$

$\hat{j}$: $\quad ma_y = 0 = N - mg\cos\theta \quad \Leftrightarrow \quad N = mg\cos\theta$

$\rightarrow$ modified free fall in rotated $\hat{i}$-direction with

$$a_x = g\sin\theta = g_{eff} \leq g$$

(note that $g_{eff} = 0$ at $\theta = 0$

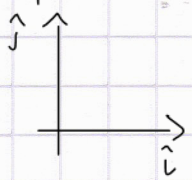
$g_{eff} = g$ at $\theta = \frac{\pi}{2}$)

constant acceleration $\rightarrow$ velocity changes linearly with time:

$$\Delta V_x = V_f - V_0 = g\,\Delta t\,\sin\theta \quad (= g_{eff}\,\Delta t)$$

(ii) w/ kinetic friction

keep rotated view



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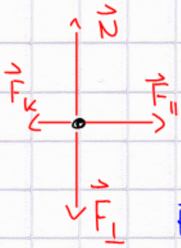


still : $\vec{F}_{grav} = F_{\parallel}\hat{i} + F_{\perp}\hat{j}$

$= mg\sin\theta\,\hat{i} - mg\cos\theta\,\hat{j}$

$f_k = |\vec{F}_k| = \mu_k|\vec{N}| = \mu_k|F_{\perp}| = \mu_k mg\cos\theta$

Newton-2 : $m a_x = F_{\parallel} - F_k = mg\sin\theta - \mu_k mg\cos\theta$

$\iff a_x = g(\sin\theta - \mu_k\cos\theta)$ (further reduced, but still constant)

$\Rightarrow \Delta V_x = g(\sin\theta - \mu_k\cos\theta)\,\Delta t$

(iii) w/ static friction : static equilibrium (as long as $|F_s| \le \mu_s N$)

$\hat{i}$: $mg\sin\theta = F_s$

$\hat{j}$: $mg\cos\theta = N$

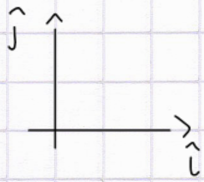
$\measuredangle$ $F_s = F_s^{max} = \mu_s N = \mu_s mg\cos\theta = mg\sin\theta$

$\iff \tan\theta = \mu_s$

if $\theta > \tan^{-1}\mu_s$ the object will begin to move !

A sled stuck in the snow

(see book, ex. 4.1)



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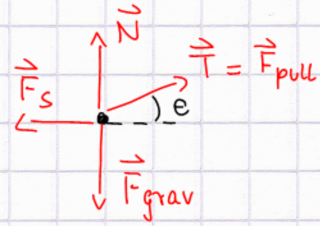


equilibrium:

$$\vec{F}_s + \vec{N} + \vec{T} + \vec{F}_{grav} = 0$$

$\hat{i}$: $\quad -F_s + T\cos\theta = 0 \quad \Leftrightarrow \quad F_s = T\cos\theta$

$\hat{j}$: $\quad N - mg + T\sin\theta = 0 \quad \Leftrightarrow \quad N = mg - T\sin\theta$

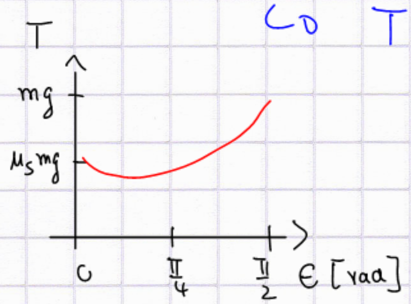
What force does one have to apply to get it out?

Overcome $F_s^{max} = \mu_s N = \mu_s (mg - T\sin\theta)$

$$\Rightarrow \quad T\cos\theta = \mu_s (mg - T\sin\theta)$$

$$\Leftrightarrow \quad T(\cos\theta + \mu_s \sin\theta) = \mu_s mg$$

$$\Rightarrow \quad T = T(\theta) = \frac{\mu_s mg}{\cos\theta + \mu_s \sin\theta}$$



find the angle θ where $T = F_{pull}$ assumes minimum:

$$T'(\theta_{opt}) = 0 \qquad \text{(necessary condition)}$$

$$T'(\theta) = \mu_s mg \frac{\sin\theta - \mu_s\cos\theta}{(\cos\theta + \mu_s \sin\theta)^2}$$

$$T'(\theta_{opt}) = 0 \quad \Leftrightarrow \quad \sin\theta_{opt} - \mu_s\cos\theta_{opt} = 0 \quad \Leftrightarrow \quad \theta_{opt} = \tan^{-1}\mu_s$$

(looks familiar!)