

BPHS 4090 (Fall 2013) - HW 2

Due Date: Sept. 25, 2013 1:00 PM

Questions

1. Show that the sum of two sinusoids is

$$\begin{aligned} A_1 \cos \omega_1 t + A_2 \cos \omega_2 t = & (A_1 + A_2) \cos \left[\frac{[\omega_1 + \omega_2]t}{2} \right] \cos \left[\frac{[\omega_1 - \omega_2]t}{2} \right] + \\ & (A_2 - A_1) \sin \left[\frac{[\omega_1 + \omega_2]t}{2} \right] \sin \left[\frac{[\omega_1 - \omega_2]t}{2} \right] \end{aligned} \quad (1)$$

This identity is useful in the context of *beating*, which arises in various aspects of biology (such as the neurophysiology of binaural beating). [Hint: Use complex exponentials to make life much easier!]

2. Let $x(t)$ be the input to an LTI system with unit impulse response $h(t)$ (i.e., the Heaviside step function $\equiv u(t)$):

$$x(t) = e^{-at} u(t) \quad (2)$$

$$h(t) = u(t) \quad (3)$$

and $a \in \mathbb{R}$. Determine the function $y(t)$ that is the convolution of x and h . Briefly explain why $y(t)$ has the form it does. Also, what do think would happen if $a \in \mathbb{C}$?

3. Consider the sequence of images shown in Fig.1.

- Briefly explain the properties of the Fourier transform of A.
- Qualitatively, explain how the Fourier transform of A and B would be different.
- Qualitatively, explain how the Fourier transform of B and C would be different.
- Qualitatively, explain how the Fourier transform of C and D would be different.
- Qualitatively, explain how the Fourier transform of C and E would be different.
- Qualitatively, explain how the Fourier transform of E and F would be different (noting that F is simply a lower resolution version of E).

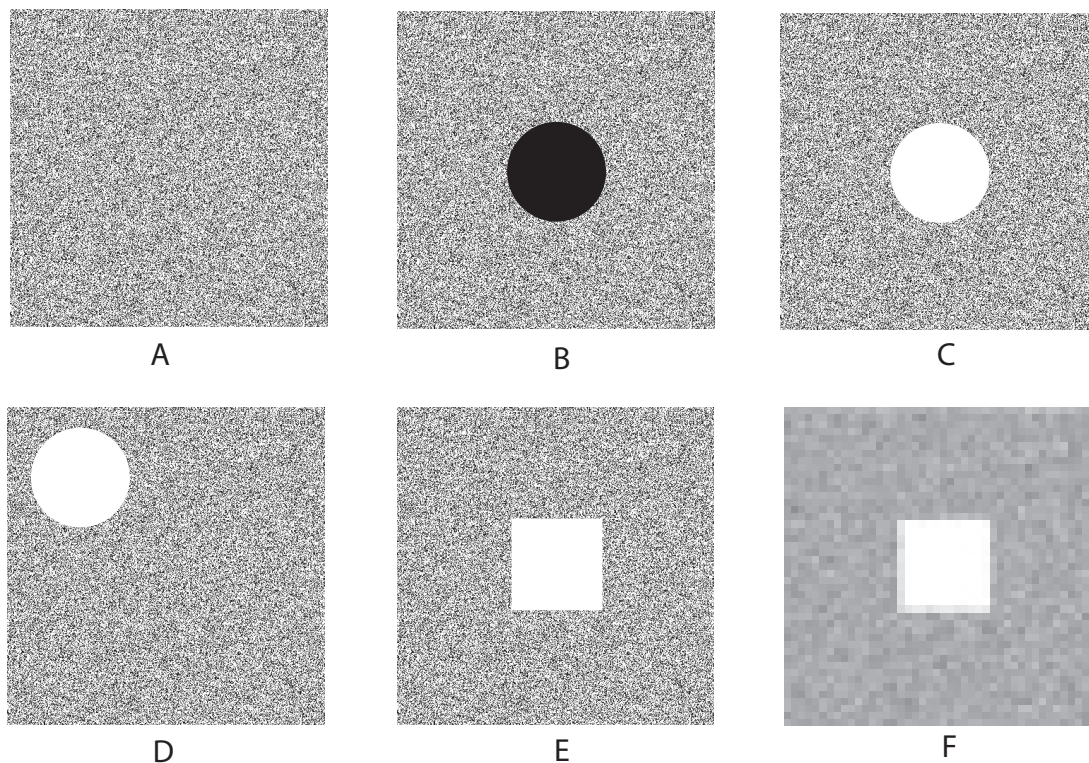


Figure 1: Simple image to consider in spectral domain.

Extra Credit: Download the raw image files of Fig. (either via the course website or via contacting the instructor). Compute the Fourier transforms and demonstrate a degree of mastery (e.g., Can you shift the circle around to an arbitrary location? Or change it's shape to a triangle?)

4. Find and read the article entitled *Introductory science and mathematics education for 21st-century biologists* by Bialek & Botstein (Science, 2004).

- a. Briefly explain how you searched for and ultimately obtained the article (i.e., how did you go about finding the information you needed).
- b. Read the article critically and write a 0.5–1 page summary detailing their argument.
- c. Write an additional 0.5–1 page summary of, based upon details of your own experience, your perceptions about what the authors claim. For example, as a biophysics student, what claims do you feel are justified? Which are not? Anything important you feel that they have left out?

5. One September 6th, 2013, the LADEE spacecraft was launched by means of a Minotaur 5 rocket from the Wallops Flight Facility in Virginia. The picture shown in Fig.2 was taken during the launch.

- a. Note the unusual feature in the image. Will this ‘feature’ achieve orbit around earth? Explain why or why not.
- b. Estimate the amount of mechanical energy (i.e., kinetic and potential) associated with this ‘feature’. Clearly explain any assumptions you make.
- c. “The first explosion of an atomic bomb was the Trinity test in New Mexico in 1945. Several years later a series of pictures of the explosion, along with a size scale, and time stamps were released and published in a popular magazine. Based on these photographs a British physicist named G. I. Taylor was able to estimate the power released by the explosion (which was still a secret at that time).” If Taylor were alive today, do you think Fig. would allow him to estimate the power released by the launching of a Minotaur 5 rocket? Explain.

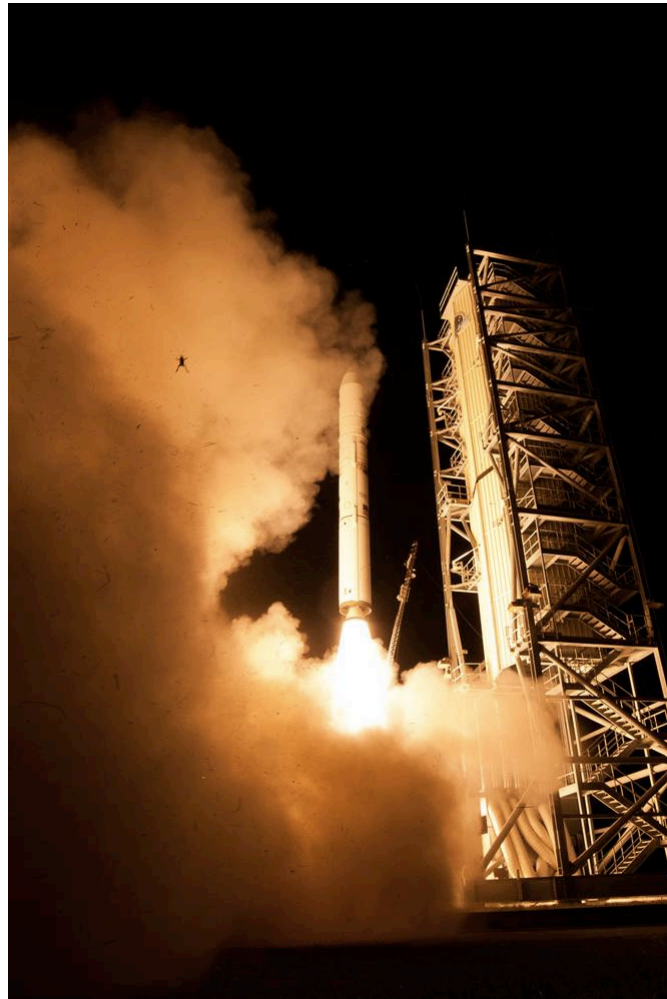


Figure 2: Space frog.