Signalling in the Internet Craze of Initial Public Offerings*

Melanie Cao
Department of Finance
Schulich School of Business
York University, Toronto,
Ontario, Canada, M3J 1P3
(email: mcao@ssb.yorku.ca)

Shouyong Shi
Department of Economics
Indiana University, Wylie Hall 105
Bloomington, IN 47405, USA
(email: shshi@indiana.edu)

This version: 2001.

Abstract

In this paper we analyze the clustering phenomenon of underpricing in initial public offerings (IPOs), where firms in a particular industry choose to issue their new shares at the same time and at great discounts. The industry consists of many firms that have private information about their own qualities (high or low) and that must raise external capital first before production. In the product market, firms compete through quality ladders, where each high-quality firm monopolizes the production of a particular variety of product. We show that self-fulfilling multiple equilibria arise. In one, no firm underprices the IPO. In the other, all high-quality firms underprice their IPOs, resulting in clustering. Moreover, the clustering is more likely to occur in economic upturns than in downturns, and in an easy credit market than in a tight market.

Keywords: Initial public offerings; Signalling; Clustering; Multiple equilibria.
JEL classifications: E44, D82.

* Corresponding author: Shouyong Shi. This paper is a revision of a previous paper, "Publicity and Clustering of IPO Underpricing", which has been presented at the Western Finance Association meeting (Idaho, 2000), ABN AMRO International Conference on IPO (Amsterdam, 2000), Pacific-Basin Capital Market/Financial Management Association Meeting (Melbourne, 2000), Northern Finance Association Meeting (Waterloo, 2000), Queen's University and University of Toronto. We thank Douglas Cumming, Deborah Lucas, Frank Milne, Jay Ritter, Ralph Winter, Kent Womack, Kathryn Wong and the conference/workshop participants for comments and discussions. Both authors gratefully acknowledge the financial support from the Social Sciences and Humanities Research Council of Canada. All errors are ours alone.

1. Introduction

In this paper we model the phenomenon that large price gains in new shares cluster sporadically in particular industries. We show that aggregate demand uncertainty in the industry's product market, together with private information about individual firms' qualities, can induce a large number of rational firms to issue their new shares at the same time and at great discounts, resulting in the clustering of subsequent price gains.

Our analysis is motivated by the Internet "craze" around 1999, in which many Internet firms issued new shares to the public, through the initial public offering (or IPO for short). Those shares experienced large price gains immediately after the IPOs, sometimes by several times as the issuing price, suggesting that the firms greatly underpriced their new shares. The clustering of such price gains in Internet IPOs contrasted sharply with the lackluster performance of concurrent IPOs in other industries.[1] It was also short-lived, as the number of underpriced Internet IPOs and the magnitude of underpricing both diminished in 2000. Why do rational firms cluster to underprice their IPOs? Why does such clustering occur in particular industries?

These questions should be interesting to macroeconomists, as well as to finance specialists. First, macroeconomists often view capital market imperfections as an important reason why monetary policies can affect real activities. By identifying the imperfections that generate the discrepancy between offer prices and market prices of new shares, we may shed light on how macroeconomic policies might stimulate the growth of new industries. Second, the finance literature, to our knowledge, has not yet provided a theoretical model to demonstrate that IPO underpricing can cluster in a market equilibrium. Instead, it focuses on the decisions of a single firm or underwriter (see the references later). To understand how clustering can be consistent with a market equilibrium, one must go beyond individual firms' behavior, and perhaps beyond the equity market as well. This task has a macroeconomic flavor.

[1] In fact, there was an increase in cancellations and withdraws from the IPO market by non-Internet firms in 1999. As the chief executive of a large dry pet food company complained, "If you look at the IPO market, there's large-capitalization activity and dot.com activity, but little else. I feel sorry for small-cap companies that are nondot.com, and which need to complete their deals." (Prial 1999)

We explore the interaction between the IPO market and the product market for a new industry that faces an uncertain aggregate demand for its products. The expectations of such demand increase with the industry's publicity, which we assume to be an increasing function of the industry-wide average price gains in IPOs. The new industry comprises of many firms, a fraction of which produce high-quality varieties of goods (or services) and others low-quality products. In order to produce, a firm must raise external capital first, either through IPO in the equity market or through other costly methods. At this stage, whether a firm is a high-quality or low-quality firm is private information. After obtaining external capital, firms compete in the goods market with quality ladders and monopolistic competition, as modelled in Grossman and Helpman (1991). That is, each high-quality firm uses its quality advantage to monopolize the production of a particular variety, while low-quality firms competitively produce those varieties whose most recent technologies have already been imitated. These varieties are complementary with each other in consumers' preferences, and so each individual firm's expected earnings increase with the aggregate demand for the industry's products. With the assumption that such aggregate demand increases with the industry's publicity, individual firms' expected earnings increase with the industry's publicity as well.

What matters for the firms' decisions in the IPO market, however, is the differential benefit in expected earnings that the industry's publicity generates to a high-quality firm, relative to a low-quality firm. We show that this differential benefit is positive and increases with the industry's publicity. This differential benefit creates the desire for high-quality firms to separate themselves from low-quality firms, by signalling quality in the IPO market. At the same time, the differential benefit increases the difficulty of signalling, because it increases low-quality firms' temptation to masquerade as high-quality firms. To signal successfully, high-quality firms may take very costly actions such as IPO underpricing. Whether they will do so depends on their expectations of the industry's publicity.

There are two self-fulfilling market equilibria, in which high-quality firms successfully separate themselves from low-quality firms.[2] In one, all high-quality firms underprice their IPOs and, in the other, no firm underprices the IPO. In the underpricing equilibrium, firms expect the industry's publicity to be high. Such expectations create a large difference in expected earnings between a high-quality and low-quality firm, which increases the temptation for low-quality firms to mimick high-quality firms in the IPO market. To signal successfully, high-quality firms underprice their IPOs. In turn, the clustering of subsequent gains in share prices increases the industry's publicity, thus fulfilling the expectations that the aggregate demand for the industry's products will be high. In the no-underpricing equilibrium, in contrast, the industry's publicity is expected to be low, and so the difference in expected earnings between a high-quality and low-quality firm will be low. There is no need to underprice IPO in this case; instead, high-quality firms signal quality by reducing the number of shares issued in IPO. In turn, the absence of large price gains in the IPOs fulfills the expectations that the industry's publicity will be low.

The clustering generates large underpricing for each high-quality firm. To emphasize this feature, we deliberately restrict the intrinsic difference between a high-quality and low-quality firm to be small, so that underpricing would not occur if there were no interactions among the firms. Even with this restriction, the clustering induces high-quality firms to underprice their IPOs by 100% in one version of our model!

Let us clarify the assumption that expected aggregate demand for the industry's products is an increasing function of the average magnitude of IPO underpricing in the industry. First, this assumption is reasonable for a very new industry, like the Internet industry, whose products are quite different from those offered by traditional businesses. Because there is little guidance to predicting the product demand for such an industry, spectacular price gains in new shares in the industry can create publicity for the industry and increase consumer awareness of the industry, thus benefiting the industry as a whole. Second, the assumption itself does not generate the result that underpriced IPOs cluster. On the contrary, it alone reduces firms' incentive to underprice.

[2]All equilibria we focus on in this paper are separating equilibria that are refined by the intuitive criterion of Cho and Kreps (1987). For each value of expected earnings, there is a unique separating equilibrium in the signalling game, but there are multiple values of expected earnings that are consistent with rational expectations.

Because individual firms take the industry's publicity as an externality, they would free-ride on the externality. So, in usual circumstances, no firm would choose to underprice its IPO.

Our model overcomes the free-rider problem because of the product market characteristics and the private information. The product market equilibrium generates the outcome that the difference in expected earnings between a high-quality and low-quality firm increases with the aggregate demand in the industry. With private information, a high-quality firm can capture this differential benefit only if it can convince the market that it is a high-quality firm. This entails costly signalling actions in the IPO market, such as underpricing.

Our paper belongs broadly to the literature of self-fulfilling, multiple equilibria (e.g., Diamond and Dybvig, 1983), but the focus on IPO is specific. In addition, the actions clustered in the equilibrium here are the attempts to signal quality. Such asymmetric information or signalling is not important for multiple equilibria in the Diamond-Dybvig model. Private information is important in the herding models (e.g., Banerjee 1992 and Bikhchandani et al. 1992), but the role is quite different from that in our model. There, herding occurs as agents choose to ignore, rather than signal, their private information.

The main contribution of our paper to the IPO literature is that we analyze why underpriced IPOs can cluster in equilibrium. Except for empirical documentations of clustering (e.g., Ritter 1984), the literature does not provide a theoretical model for clustering, because it focuses on an individual firm's or underwriter's underpricing decisions.[3] For example, Allen and Faulhaber (1989), Welch (1989), and Grinblatt and Hwang (1989), and Tambanis and Bernhardt (1999) use the signalling model to examine IPO underpricing. Rock (1986) emphasizes information asymmetry in a different way, arguing that the winner's curse forces a firm to underprice IPO in order to attract uninformed investors. Others attribute underpricing to underwriters who try to build good reputations through the price gains (e.g., Beatty and Ritter 1986, and Benveniste and Spindt 1989), to a firm's concern for liquidity in the secondary market (e.g., Mauer and Sen-

[3] See Michaely and Shaw (1994) for an extensive list of references and empirical works. Loughran and Ritter (2000) provide some recent references.

bet 1992), or to behaviors that are not Bayesian rational (Loughran and Ritter 2000). Another related contribution of our analysis is that we link the IPO market equilibrium to the equilibrium in the industry's product market. The quality-ladder framework generates product demand complementarity, which makes it rational for firms to cluster their actions.

We use a signalling model to describe the IPO decisions and abstract from the institutional features like underwriters.[4] When a firm underprices IPO, it transfers a part of its value to investors free of charge. One may ask why firms choose this specific way to signal, rather than other "money-burning" actions such as advertisement. One reason is that other money-burning actions entail current resources which a new firm may not have. In contrast, signalling by underpricing IPO entails only expected future earnings. In comparison with advertisement, specifically, IPO underpricing is superior for additional reasons. First, when the entire industry is new, advertisement may not be as effective as the hard evidence of IPO price gains. Second, advertisement must be monitored in order to be credible, while IPO price gains are publicly observed.[5]

2. The Model

2.1. Industry Uncertainty and Private Information

As argued in the introduction, a new industry like the Internet industry faces aggregate uncertainty in the product demand, and investors' expectations on the industry are susceptible to the industry's IPO performance. To capture this idea, let Y be consumers' aggregate expenditure on the industry's products and D_a the industry-wide average price gain in new shares immediately

[4] Underwriters' concern for reputation may be a good explanation for isolated cases of underpricing, but not for the clustering of underpricing. When underpriced IPOs cluster, price gains are prevalent across underwriters, concentrated in a particular industry, and short-lived. In contrast, underwriters who are motivated by reputation should underprice IPOs in all industries, rather than a particular industry, and for a long period of time (since reputation needs time to build), rather than for a short period of time like 1999.

[5] Signalling models have been criticized on the basis that post-IPO earnings do not seem to have much explanatory power for the underpricing magnitude (see Michaely and Shaw, 1994). For two reasons we view this as inconclusive evidence against the signalling model. First, the signalling model predicts that the magnitude of underpricing depends positively only on the part of future earnings that is private information prior to IPO. Most of the empirical tests do not distinguish this part of future earnings from the part that is publicly expected prior to IPO. Second, because post-IPO performances depend on post-IPO investment strategies that may not be foreseen at the time of IPO, such performances may not be good indicators of the firms' conditional expected earnings at the time of IPO. Most of the empirical tests do not control for such a diversity in post-IPO investment strategies.

after IPO. Expected aggregate demand for the industry's products is as follows:

$$E(Y|D_a) = Y_0 + \rho_a D_a, \quad \rho_a > 0. \tag{2.1}$$

There are two justifications for $\rho_a > 0$. First, investors may spend some of the price gains from Internet IPOs on Internet goods. Second, the clustering of large price gains in new shares creates publicity for the industry and, as consumers become more aware of the industry, they may switch some expenditure from traditional goods to the Internet goods. For example, Internet firms that sell books, auction goods, or provide market information on Internet compete against businesses that organize such activities in traditional ways. If these firms' IPOs have large price gains, the publicity may induce customers to switch from traditional firms to these new firms, e.g., switching from buying books in neighborhood bookstores to Internet book-selling firms.

The assumption (2.1) is important to our results in the sense that it links the industry's aggregate IPO performance and the aggregate demand in the product market. It would be erroneous, however, to think that we are assuming the result of clustering by imposing this assumption. As explained in the introduction, the assumption (2.1) itself would generate the free-rider problem and lead to the opposite result that no firm would choose to underprice the IPO. We will establish this result formally in Proposition 2.2 later.

The industry produces a continuum of varieties of goods and the firms are on two quality ladders. Let $i \in [0, 1]$ be the index of varieties and $\alpha \in (0, 1)$ the fraction of high-quality firms in the industry. Each variety in the sub-interval $[\alpha, 1]$ is produced by low-quality firms, whose technology can be easily imitated, and so there is perfect competition for the production of such varieties. In contrast, each variety in the sub-interval $[0, \alpha)$ is produced by a high-quality firm, which uses its advanced technology to drive out competition. Delaying the description of "quality" to the next subsection, we use $x = x_H > 1$ to indicate high-quality and $x = x_L = 1$ low-quality. To produce, a firm must have an amount of capital $k_0 > 1$, which is the same for all firms. However, a firm is endowed with only $(k_0 - 1)$ amount of capital and hence must raise one unit of external capital, through initial public offering and/or alternative financing methods.

At the time of raising external capital, a firm's quality is private information. The market belief about each firm prior to IPOs coincides with the population statistics of firms, i.e., the firm is of high-quality with probability α and of a low-quality with probability $1-\alpha$. In the IPO market, a firm chooses the offer price s and the number of shares f to be offered. Normalize the total number of a firm's shares to 1, so that $f \in (0,1]$. The firm's original owners keep $1-f$ shares. The market price of shares is p. The gain to IPO investors is $d \equiv p-s$ per share. The firm underprices IPO if $d>0$. The IPO revenue is $q \equiv sf$. If $q<1$, the firm finances the remainder through alternative methods such as venture capital, loans, etc. The expected cost of alternative funds is $\left(1+bx^{-1}\right)(1-q)$, where $b>0$ is a constant. Thus, for each unit of alternative funds, the additional expected cost is bx^{-1}, which is decreasing in the firm's quality.[6]

The sequence of actions is as follows. First, firms go to the IPO market to issue shares, and all firms do so at the same time (see Section 5 for sequential decisions). Second, if a firm's IPO revenue falls short of the required amount (1 unit), the firm seeks alternative financing. Third, firms combine capital with labor to produce and compete in the product market. After paying the labor cost, each firm repays the alternative funds first and then shareholders.

We want to clarify a few points about the alternative financing methods. First, by assuming that the expected cost of alternative funds is a decreasing function of the firm's quality, we do not mean that alternative financiers know the firm's quality perfectly. Rather, the financiers' knowledge of the firm is imperfect and positively correlated with the firm's true quality. For example, the financiers may screen the firm and determine the loan rate according to the screening outcome. If the screening technology yields a noisy signal that is positively correlated with the firm's quality, then the expected cost of the funds is a decreasing function of the firm's true quality, although the financiers do not know the firm's quality. Second, it is not necessary (but convenient) to assume that firms go to the IPO market first and then to alternative financiers. If, instead, a firm seeks alternative funds first, then the IPO investors can infer the financiers'

[6] The linear cost function keeps the analysis simple, but all analytical results in this paper hold for a more general cost function $(1+b/x)C(1-q)$ that satisfies $C(0)=0$, $C'(0)\geq 1$ and $C''>0$.

screening outcome by observing the cost of the firm's alternative funds. This alleviates, but does not solve, the asymmetric information problem in the IPO market, because the alternative financiers' screening outcome is not a perfect indicator of the firm's true quality. With positive probability a high-quality firm can be wrongly labeled as low-quality by the alternative financiers, in which case the firm may still find it useful to signal its true quality through the IPO actions.[7]

2.2. Product Market and Firms' Earnings

Because a firm's IPO decision depends on the expectations of the firm's earnings, we analyze the equilibrium in the product market first. Suppose that high-quality firms have successfully separated themselves through the IPO activities, an outcome we will establish later. Then, a firm's quality is public information in the product market. The capital cost, which has been determined endogenously in the IPO market, is denoted k_L for a low-quality firm and k_H for a high-quality firm. Note that $k_j \geq k_0 > 1$, for $j = H, L$.

In the product market, a high-quality firm is the technological leader in the production of a particular variety. We capture this technological advantage by assuming that a high-quality firm needs less labor to produce than a low-quality firm (see Grossman and Helpman 1991). The labor input required to produce $c(i)$ units of variety i is

$$l = c(i)/c_0 \text{ if } i \in [0, \alpha), \text{ and } l = c(i)/(\beta c_0) \text{ if } i \in [\alpha, 1],$$

where $0 < \beta < 1$ and $c_0 > 0$. Thus, each variety in $[0, \alpha)$ is produced by a high-quality firm and each variety in $[\alpha, 1]$ by a low-quality firm. A high-quality producer of a variety has a technological advantage for that variety (of a factor $1/\beta$) over its potential low-quality imitators.

A representative consumer has the following preferences over the varieties:

$$\exp\left(\int_0^1 \ln[c(i)]di\right),$$

[7]Empirical evidence seems to support this argument. For example, James and Wier (1990) and Slovin and Young (1990) find that IPOs of firms with previously established borrowing relationships can still experience IPO underpricing, although they may underprice by less than other IPOs.

where $c(i)$ is the amount of consumption of variety i. Let $\pi(i)$ be the price of variety i, measured relative to non-Internet goods. Then, the consumer's maximization problem is

$$\max \ \exp\left(\int_0^1 \ln[c(i)]di\right) \text{ subject to } \int_0^1 \pi(i)c(i)di \leq Y.$$

Solving this problem, we have the following demand curve for variety i:

$$\pi(i) = Y/c(i), \text{ for all } i \in [0,1].$$

Facing the demand curve, a firm maximizes net profit. Let l_j be the labor input of a type-j firm, $j = H, L$, and normalize the wage rate to 1.[8] A type-j firm's net profit is $\pi(i)c(i) - l_j - k_j$. Because the technology to produce a low-quality variety can be readily imitated, net profit is zero for such a firm. In contrast, a high-quality firm monopolizes the production of a particular variety, by setting the price to be just low enough to prevent low-quality imitators from entering. In Appendix A we solve these profit-maximizing problems and establish the following Lemma.

Lemma 2.1. *In the goods-market equilibrium, the earnings of a firm that can be distributed to its lenders and shareholders are r_L for a low-quality firm and r_H for a high-quality firm, where*

$$r_L \equiv \pi(i)c(i) - l_L = k_L, \tag{2.2}$$

$$r_H \equiv \pi(i)c(i) - l_H = (1-\beta)Y + \beta k_L. \tag{2.3}$$

An important result here is that a high-quality firm's earnings are an increasing function of the aggregate product demand, Y, while a low-quality firm's earnings are determined entirely by the capital cost k_L. Thus, an expected high industry's publicity widens the difference in expected earnings between a high-quality and low-quality firm. To express this feature formally, denote the expected earnings of a low-quality firm by R_0 and R_H of a high-quality firm, conditional on IPO activities. Let D be the amount with which a representative high-quality firm underprices its

[8] This can be delivered by the following structure. Suppose that traditional goods are produced using only labor and the production technology is linear in labor input, normalized to l. Then, net profit of a firm in such a traditional sector is $l - wl = (1-w)l$, where w is the wage rate. Perfect competition in this sector yields $w = 1$.

IPO. Because a low-quality firm does not underprice its IPO, as shown later, the average amount of underpricing in the industry is $D_a = \alpha D$. (2.1), (2.2) and (2.3) then imply

$$R_0 \equiv E(r_L|D) = k_L \ (\geq k_0 > 1), \quad R_H \equiv E(r_H|D) = R_0 x_H + \rho D, \tag{2.4}$$

where $\rho \equiv (1-\beta)\alpha\rho_a > 0$ and $x_H \equiv \beta + (1-\beta)Y_0/k_L$. Thus, a high-quality firm's expected earnings indeed respond more positively to the industry's publicity D.

To separate the terms in expected earnings that are unrelated to the industry's publicity, we call R_0 the intrinsic earnings of a low-quality firm and $R_0 x_H$ of a high-quality firm. With this terminology, we simply refer to x as a firm's quality, where $x = x_H$ for a high-quality firm and $x = 1$ for a low-quality firm. Assume $Y_0 > k_L$, so that $x_H > 1$. Also, assume $\rho < 1$.

2.3. Initial Public Offering

Let us analyze the IPO decision of an individual firm. Because the firm takes the industry's publicity D as given, it takes expected earnings (R_0, R_H) as given. Express the firm's IPO decision as $a \equiv (f, q)$, rather than (f, s). Let $I \in [0, 1]$ be the posterior belief (probability) in the market that the firm is of high-quality, conditional on all firms' IPO activities. Then the market expects the firm's earnings to be:

$$R_I \equiv E(r|I) = R_H I + R_0(1 - I). \tag{2.5}$$

The expected return to the original owners is as follows:[9]

$$V(f, q; R_I, x) \equiv (1 - f)\left[R_I - \left(1 + bx^{-1}\right)(1 - q)\right]. \tag{2.6}$$

The firm chooses a to maximize V. Note that the firm knows its own quality x.

Investors do not know the firm's quality at the time of IPO, and so they are concerned with the expected rate of return to shares. In equilibrium, this rate of return must be equal to the risk-free rate of return. To simply algebra, we normalize the gross, risk-free rate of return to 1.

[9]Throughout this paper the payoff to a firm refers to the payoff to the original owners of the firm after IPO, rather than the payoff to all shareholders.

Then, the market price of IPO shares must be equal to the expected return per share. Let p_I be the market price of a share when the posterior belief about the firm is I. Then,

$$p_I = R_I - \left(1 + bE_I x^{-1}\right)(1 - q), \tag{2.7}$$

where $E_I x^{-1} \equiv x_H^{-1} I + (1 - I)$. For investors to participate in IPO, the offer price cannot exceed the market price. That is,

$$0 \leq s = q/f \leq p_I. \tag{2.8}$$

A high-quality firm may want to signal its quality by refraining from a high IPO revenue, through underpricing and/or issuing fewer shares in IPO. Because a low-quality firm may mimic, for successful separation a high-quality firm must have greater incentive to signal than a low-quality firm. The well-known single-crossing property is then necessary (see Fudenberg and Tirole 1993, p.259), which is satisfied by the payoff function $V(f, q; R, x)$ in the following forms:

$$\frac{\partial}{\partial x}\left(-\frac{\partial V/\partial R}{\partial V/\partial q}\right) < 0; \quad \frac{\partial}{\partial x}\left(-\frac{\partial V/\partial R}{\partial V/\partial f}\right) < 0. \tag{2.9}$$

The first condition states that, for a fixed number of shares issued in IPO, a high-quality firm is willing to reduce the IPO revenue by more than does a low-quality firm in order to receive an increase in the expectations on earnings "rewarded" by the market. The second condition states that, for a fixed IPO revenue, a high-quality firm increases the number of shares issued in IPO by less than does a low-quality firm in the event of an increased expected earnings.

Assumption 1. 1A. *A high-quality firm, if its quality is publicly known, can make a positive return even when all external capital comes from alternative funds, i.e.,* $R_0 x_H > 1 + b/x_H$.

1B. *A low-quality firm, if its quality is publicly known, cannot make a positive return when all external capital comes from alternative funds, i.e.,* $R_0 < 1 + b$.

1C. *The intrinsic earning difference between high-quality and low-quality firms is not too large, i.e.,* $R_0(x_H - 1) < b$.

Assumption 1A gives a high-quality firm some ability to signal its quality: Since it makes a positive return even with 100% non-equity funds, it can reduce the IPO revenue to signal its high quality. But the signalling attempt may or may not require underpricing. Assumption $1B$ makes it desirable for a low-quality firm to finance its investment through equity if its quality is publicly known. Since the quality is not publicly known, however, a low-quality firm may try to use non-equity financing to mimic a high-quality firm.

Assumption $1C$ highlights a difference between our model and previous signalling models of IPOs (e.g., Allen and Faulhaber 1989, Welch 1989, and Grinblatt and Hwang 1989). This deliberate assumption ensures that underpricing does not occur in these models (see Section 4). The assumption also seems realistic for Internet firms, because the intrinsic difference between those firms does not seem large at the beginning.

2.4. The Case of Public Information

Let us first analyze the IPO decision in the case where the firm's quality is public information. This analysis not only provides a baseline with which we can compare the results in the case of asymmetric information, it also establishes our earlier claim that the assumption (2.1) itself does not generate the clustering of underpriced IPOs.

When a firm is known to be of low-quality, the best choice in the equity market is to obtain all external capital by issuing shares at the full price. With this strategy, $q = 1$, $f = 1/R_0$, and the payoff to the firm is $(R_0 - 1) > 0$. If this firm chooses $q < 1$, instead, the market price of the firm's share is $p_L = R_0 - (1 + b)(1 - q)$. Since $f \geq q/p_L$, the payoff to the firm satisfies

$$\begin{aligned} V(f, q; R_0, x_L) &\leq (1 - q/p_L)\left[R_0 - (1 + b)(1 - q)\right] \\ &= R_0 - (1 + b)(1 - q) - q. \end{aligned}$$

The last expression is maximized at $q = 1$ and so the payoff is less than $(R_0 - 1)$ if $0 < q < 1$. The payoff is also less than $(R_0 - 1)$ if the firm underprices its IPO, which results in $f > q/p_L$ and hence strict inequality in the above expression.

Similarly, if a firm is known to be of high-quality, the best choice is to obtain all external capital by issuing shares at the full price and free-ride on the industry's publicity. Therefore, we have the following proposition:

Proposition 2.2. *If qualities are public information, no firm underprices IPO in equilibrium, despite the influence of the industry's publicity on individual firms' expected earnings.*

3. Signalling Equilibrium

In this section we characterize a firm's strategy under private information, taking the industry's publicity D (and hence R_H) as given. We refer to this best response of a single firm, together with the market belief, as a signalling equilibrium. Of course, D and R_H must also be determined in a market equilibrium, which analysis will be delayed to the next section. For given R_H, a Bayesian signalling equilibrium consists of market beliefs I and the firm's decisions (f, q) that satisfy the following conditions: (i) Given the beliefs, the firm's decisions maximize the payoff $V(f, q; R_I, x)$; and (ii) With the firm's choices, the beliefs are rational according to Bayes updating.

As is well known, there is a large set of such equilibria, because the beliefs off the equilibrium path are arbitrary. For example, consider a pooling action $a_0 \equiv (f_0, q_0)$. Since the market does not gain any new information about a firm's quality from observing this pooling action, the market's belief after observing a_0 is the same as the prior, i.e., $I = \alpha$. Denote a firm's payoff from the pooling action as $V^0(x) \equiv V(f_0, q_0; R_\alpha, x)$, where x is the firm's quality (recall $x_L = 1$). Suppose that the action and the belief satisfy the following conditions:

$$f_0, q_0 \in [0, 1]; \tag{3.1}$$

$$q_0/f_0 \leq p_\alpha = R_\alpha - \left(1 + bE_\alpha x^{-1}\right)(1 - q_0); \tag{3.2}$$

$$V^0(1) \geq R_0 - 1. \tag{3.3}$$

The action a_0 satisfying the above conditions is a Bayesian signalling equilibrium, supported by the belief that any deviation from this action is made by a low-quality firm.[10] Condition (3.1) is self-explanatory and (3.2) requires the offer price to be at most the market price. Condition (3.3)

[10] To see this, suppose that a high-quality firm deviates from a_0. Then, according to the particular belief, this firm will be perceived as a low-quality firm. In this case, the best action is $(q, f) = (1, 1/R_0)$, which generates a lower payoff than the pooling action a_0 under (3.3).

requires a low-quality firm's payoff in the pooling equilibrium to be at least that from revealing the firm's type, in which case the best actions are $(q, f) = (1, 1/R_0)$ and the payoff is $R_0 - 1$.

We use the intuitive criterion of Cho and Kreps (1987) to refine the equilibrium set, and so the term "equilibrium" in this paper stands for an equilibrium that satisfies this criterion. The Cho-Kreps intuitive criterion is a restriction on the beliefs off the equilibrium path. To describe this restriction in our model, consider a deviation $(f, q) \neq (f_0, q_0)$ that satisfies the following conditions. First, the deviation is feasible for a high-quality firm, i.e., $f, q \in [0, 1]$ and the offer price does not exceed the implied market price:

$$0 \leq q/f \leq p_H = R_H - (1 + bx_H^{-1})(1 - q). \tag{3.4}$$

Second, if a low-quality firm makes the same deviation, it gets less than in the pooling equilibrium even when it is viewed as a high-quality firm:

$$(1 - f)\left[R_H - (1 + b)(1 - q)\right] < V^0(1). \tag{3.5}$$

Third, the deviation generates a higher payoff to the high-quality firm than in the pooling equilibrium if the firm is viewed as a high-quality firm as a result of the deviation:

$$(1 - f)\left[R_H - (1 + bx_H^{-1})(1 - q)\right] > V^0(x_H). \tag{3.6}$$

Actions that satisfy (3.4), (3.5) and (3.6) are "credible" deviations by a high-quality firm. A low-quality firm will not take make such deviations even when it is given the benefit of doubt and viewed as a high-quality firm. In contrast, a high-quality firm gains by such deviations, if the market views the deviator as a high-quality firm. Observing these deviations, the market should intuitively interpret the deviator as a high-quality firm, as argued by Cho and Kreps (1987).

To see how this restriction helps refining the equilibrium set, let us first rewrite (3.4) and (3.5). Under Assumption $1A$, (3.4) can be rewritten as $f, q \in [0, 1]$ and

$$f \geq S_H(q) \equiv q \Big/ \left[R_H - (1 + bx_H^{-1})(1 - q)\right]. \tag{3.7}$$

To rewrite (3.5), define a critical level:

$$Q_1 \equiv 1 - \frac{R_H - V^0(1)}{1+b}. \tag{3.8}$$

Since $V^0(1) \leq R_\alpha - 1$ (see Appendix B), $Q_1 < 1$. If either $Q_1 < 0$ or $q \leq Q_1$ then (3.5) is satisfied for all $f \in [0,1]$. For $q \geq \max\{0, Q_1\}$, (3.5) can be rewritten as

$$f > IND_L(q) \equiv 1 - V^0(1)\,/[R_H - (1+b)(1-q)]\,. \tag{3.9}$$

Similar to (3.7), (3.2) can be rewritten as follows:

$$f \geq S_\alpha(q) \equiv q \Big/ \left[R_\alpha - \left(1 + bE_\alpha x^{-1}\right)(1-q)\right]. \tag{3.10}$$

Figures 1a and 1b depict the curves $f = S_H(q)$, $f = IND_L(q)$ and $f = S_\alpha(q)$ for the two cases $Q_1 < 0$ and $Q_1 > 0$, respectively. The curve $f = S_H(q)$ is the full-price curve for a high-quality firm, above which underpricing occurs; Similarly, the curve $f = S_\alpha(q)$ is the full-price curve in a pooling action. The curve $f = IND_L(q)$ is the set of actions to which a deviation by a low-quality firm generates the same payoff as the pooling action (f_0, q_0) when the firm is viewed as a high-quality firm after the deviation. Actions above the curve $f = IND_L(q)$ generate strictly lower payoffs to a low-quality firm even if the firm is viewed as a high-quality firm after a deviation to such actions. Thus, the shaded area in each diagram is the set of actions that satisfy (3.7) and (3.9) (i.e., (3.4) and (3.5)). The Cho-Kreps criterion requires that the market view any such deviation as coming from a high-quality firm and attach a belief $I = 1$ to the deviator.

Figures 1a and 1b here.

A high-quality firm should consider only deviations that maximize its payoff. For any deviation in the shaded area that is not the best, a further deviation to the best action does not change the market's belief ($I = 1$) under the Cho-Kreps criterion but improves a high-quality firm's payoff. To find the best credible deviation from a pooling action, we need to know the properties of $S_H(q)$ and $IND_L(q)$, which are summarized in the following lemma and proved in Appendix B:

Lemma 3.1. *(i) Under Assumptions $1A - 1B$, $IND'_L(q) > 0$ and $IND''_L(q) < 0$ for all $q > Q_1$; $S'_H(q) > 0$ and $S''_H(q) < 0$ for all $q > 0$. (ii) If $Q_1 < 0$, then $IND_L(q) > S_H(q)$ for all $q \geq 0$; If $Q_1 \geq 0$, then there is a unique solution to $IND_L(q) = S_H(q)$ in the range $q \geq Q_1$, denoted Q_A, such that $IND_L(q) > S_H(q)$ if and only if $q > Q_A$. (iii) A high-quality firm's payoff is an increasing function of q along $f = S_H(q)$ and a decreasing function of q along $f = IND_L(q)$.*

While (i) and (ii) are mechanical, the property (iii) is important and can be explained as follows. A high-quality firm's payoff is an increasing function of q along the full-price curve $f = S_H(q)$ because, as the firm raises a higher revenue through IPO without underpricing, the firm economizes on the cost of alternative funds and so expected profit increases. To explain why a high-quality firm's payoff is a decreasing function of q along $f = IND_L(q)$, recall that a high-quality firm's desire to increase the number of shares issued in IPO is weaker than a low-quality firm's (see (2.9)). As actions move upward along the curve $f = IND_L(q)$, the IPO revenue increases and such actions are increasingly more enticing to a low-quality firm. To keep a low-quality firm indifferent between these actions and the pooling action, the number of shares issued in IPO must increase more sharply than it is desirable to a high-quality firm.

Lemma 3.1 implies that the best deviation by a high-quality firm from the supposed pooling equilibrium is arbitrarily close to and above the action depicted by point A, in Figure 1a if $Q_1 < 0$ and Figure 1b if $Q_1 > 0$. To see this, note that the firm's payoff increases when actions move southeast in Figures 1a and 1b, and so the best deviations are located arbitrarily close to and above the lower boundaries of the shaded areas. Moreover, since a high-quality firm's payoff is an increasing function of q along the full-price curve $f = S_H(q)$ and a decreasing function of q along the curve $f = IND_L(q)$ (see Lemma 3.1), the best deviation is arbitrarily close to and above point A in Figure 1a (or Figure 1b). The limit of this deviation is point A, described by:[11]

$$(f_b, q_b) \equiv \begin{cases} \left(1 - \frac{V^0(1)}{R_H - 1 - b}, \quad 0\right), & \text{if } Q_1 \leq 0 \\ (S_H(Q_A), \quad Q_A), & \text{if } Q_1 > 0. \end{cases} \tag{3.11}$$

For a high-quality firm to deviate to point A, (3.6) must also hold. The following lemma, proved in Appendix C, describes the necessary and sufficient condition for (3.6):

Lemma 3.2. *From any pooling action* (f_0, q_0)*, a deviation by a high-quality firm to* (f_b, q_b) *increases the payoff iff* $(1 - f_b)R_H > (1 - f_0)R_\alpha$.

When the deviation (f_b, q_b) dominates the pooling action under the Cho-Kreps intuitive criterion, there is no pooling equilibrium and so the best action for a low-quality firm is $(f, q) = (1/R_0, 1)$, yielding a payoff $R_0 - 1$. Replacing $V^0(1)$ by $R_0 - 1$, the condition $Q_1 \leq 0$ becomes $R_H \geq R_0 + b$. Also, denote the corresponding values of (f_b, q_b) by (f^*, q^*). Thus, for $R_H \geq R_0 + b$, Figure 1a applies and

$$f^* = 1 - \frac{R_0 - 1}{R_H - (1 + b)}, \qquad q^* = 0. \tag{3.12}$$

For $R_H < R_0 + b$, Figure 1b applies, in which case the action (f^*, q^*) solves:

$$\begin{cases} f^* = S_H(q^*); \\ \frac{q^*}{R_H - (1 + b x_H^{-1})(1 - q^*)} = 1 - \frac{R_0 - 1}{R_H - (1 + b)(1 - q^*)}. \end{cases} \tag{3.13}$$

We have the following propositions (see Appendix C for a proof):

Proposition 3.3. *Under the Cho-Kreps intuitive criterion, a separating signalling equilibrium exists. In this equilibrium, a high-quality firm takes actions* (f^*, q^*)*, characterized by (3.12) when* $R_H - R_0 \geq b$ *and by (3.13) when* $R_H - R_0 < b$*. A low-quality firm takes actions* $(f, q) = (1/R_0, 1)$*, which entail no underpricing. For any given* R_H*, this is the only separating equilibrium. If* $\alpha < \underline{\alpha}$*, where* $\underline{\alpha} \in (0, 1)$ *is defined in appendix C, there is no pooling equilibrium.*

[11] In the borderline case $Q_1 = 0$ (where point A coincides with the origin of the plane), the best deviation is $f = \varepsilon > 0$ and $q = 0$, where ε is sufficiently small. Since this case involves underpricing, it can be grouped with the case $Q_1 < 0$.

We focus on the unique separating equilibrium by restricting $\alpha < \underline{\alpha}$.[12] A high-quality firm has a preference over the two ways to signal and separate from a low-quality firm, reducing the number of issues and underpricing. Although both reduce the IPO revenue, the first method is preferable when the difference in expected earnings relative to a low-quality firm is small. By not underpricing IPO, the original owners can keep a large stake of the firm and its future earnings. If they underprice, instead, they give up a larger number of shares to the public and hence a larger claim on future earnings. Despite this costly nature of underpricing, a high-quality firm chooses to underprice IPO because reducing the number of issues alone is not sufficient for signalling successfully when other firms underprice their IPOs. Even by reducing f to zero the firm can only signal a differential in expected earnings of b.[13] For expected earnings higher than this level, the firm must sacrifice even more in order to prevent a low-quality firm from mimicking, and this entails underpricing. When the firm underprices IPO, the number of underpriced shares increases with the level of expected revenue that the firm wants to signal. That is, f increases with R_H in the underpricing region.

The number of shares issued in IPO has a U-shaped relationship with the firm's expected earnings, as depicted in Figure 2. When a high-quality firm's expected earnings increase from low levels, the number of shares issued to the public decreases, while IPO is at the full market price. This continues until the number of shares issued to the public reaches a minimum, which is zero in this version of the model. Then the number of IPO shares increases with the expected earnings (see subsection 5.1 for more discussions).

Figure 2 here.

[12] Some pooling equilibria survive the Cho-Kreps refinement under certain conditions because the extent to which a high-quality firm can signal is limited by the requirement that the IPO revenue be non-negative. When the expected earning of a high-quality firm is sufficiently higher than that of a low-quality firm, the high-quality firm must incur a sufficiently high signalling cost in order to prevent a low-quality from mimicking. This becomes difficult when there is a lower bound (0) on the IPO revenue and so some pooling equilibria with a small IPO revenue can survive.

[13] This is obtained by setting $(f, q) = (0, 0)$ and $V^0(1) = R_0 - 1$ in the equality form of (3.9).

4. Market Equilibrium and Clustering

A symmetric market equilibrium is a pair (d, D) such that d is the best response of a high-quality firm to D, given implicitly by Proposition 3.3, and that $d = D$. Once D is determined, R_H is also determined. We show that there exist multiple, self-fulfilling market equilibria. This type of multiplicity is different from the usual multiplicity in signalling games, because the signalling game here has a unique separating equilibrium for any given R_H.

To begin, denote

$$D_0 \equiv \left[b - R_0 \left(x_H - 1\right)\right] / \rho. \tag{4.1}$$

Then $D_0 > 0$ (Assumption $1C$). Also, $R_H - R_0 \geq b$ if and only if $D \geq D_0$. We can then rewrite a high-quality firm's best response to D in Proposition 3.3 as follows:

$$d = \begin{cases} 0, & \text{if } D < D_0 \\ p_H = \rho D + R_0 x_H - 1 - b x_H^{-1}, & \text{if } D \geq D_0. \end{cases} \tag{4.2}$$

Imposing the equilibrium requirement $d = D$ on (4.2), we can verify the following proposition (the proof is straightforward and omitted):

Proposition 4.1. *Define* $\underline{\rho} \in (0, 1)$ *as follows:*

$$\underline{\rho} \equiv \frac{b - R_0(x_H - 1)}{b(1 - x_H^{-1}) + R_0 - 1}. \tag{4.3}$$

Under Assumptions $1A - 1C$, *there is a market equilibrium for all* $0 \leq \rho < 1$, *where no firm underprices IPO. A market equilibrium, where all high-quality firms underprice IPOs, exists if and only if* $\underline{\rho} \leq \rho < 1$. *Thus, when* $0 \leq \rho < \underline{\rho}$, *only the no-underpricing equilibrium exists; when* $\underline{\rho} \leq \rho < 1$, *the underpricing equilibrium and the no-underpricing equilibrium co-exist. In the underpricing equilibrium, the amount of underpricing increases with* ρ.

Figure 3 depicts the case $\underline{\rho} < \rho < 1$. The underpricing "curve" depicts the best response (4.2), The no-underpricing equilibrium is at point EN and the underpricing equilibrium is at point EU. In both equilibria high-quality firms successfully separate themselves from low-quality firms. They do so in the no-underpricing equilibrium by reducing the number of issues only and, in the underpricing equilibrium, by underpricing.

Figure 3 here.

The above proposition has several noteworthy aspects. First, when $0 \leq \rho < \underline{\rho}$, the no-underpricing equilibrium is the only equilibrium. In this case, the level D_0 is large and the underpricing curve lies below the 45-degree line for all $D > 0$. Thus, no firm underprices its IPO if the industry's publicity has only a weak effect on the industry's expected product demand. Similarly, there is no underpricing if there is only one firm in the industry.[14]

Second, the no-underpricing equilibrium exists for all $\rho \in [0, 1)$. Even when the industry's publicity has a strong effect on the industry's expected product demand (i.e., when $\rho > \underline{\rho}$), firms will not underprice if they expect that other firms will not underprice. With such expectations, the difference in expected earnings between a high-quality and a low-quality firm is small, as maintained by Assumption $1C$. Then low-quality firms' temptation to mimic is weak, in which case a high-quality firm can separate itself out by reducing the number of issues alone. The absence of underpricing in turn supports the low expectations of the industry's publicity.

Third, the two equilibria both exist when the externality is strong (i.e., when $\underline{\rho} \leq \rho < 1$). The coexistence is an outcome of self-fulfilling expectations. We have already explained why a firm will not underprice if it expects that other firms will not underprice. On the other hand, if a high-quality firm expects that other high-quality firms will underprice, the difference in expected earnings between high-quality and low-quality firms is large, due to the influence of the industry's publicity. A low-quality firm's temptation to mimic is strong in this case and so a high-quality firm must underprice in order to separate itself from a low-quality firm. Since all high-quality firms underprice IPOs in this case, the clustering of underpriced IPOs in turn fulfills the expectations that the industry's publicity is high. The coexistence of the no-underpricing equilibrium with the

[14] As noted before, Assumption $1C$ is important for this result. When the intrinsic earning difference between a high-quality and a low-quality firm is large enough to violate Assumption $1C$, then $\underline{\rho} < 0$ and there is a need for a high-quality firm to underprice anyway. In fact, only the underpricing equilibrium exists in this case.

underpricing equilibrium illustrates the fragility of the clustering of underpriced IPOs.

Finally, the model is capable of producing large underpricing. In the underpricing equilibrium, high-quality firms offer their shares free of charge! When expected industry's publicity passes over the critical level D_0, the offer price drops to 0 and the percentage of discount that a high-quality firm offers to IPO investors jumps from 0 to 100%.[15] The large underpricing resembles those observed in some Internet IPOs in 1999. Considering that the intrinsic difference between high-quality and low-quality firms is small (Assumption $1C$), the large magnitude of underpricing is remarkable. With the same restriction on the earning differential, there is no underpricing in previous signalling models. To understand this difference between our model and previous models, it is important to note that the market price of shares is endogenous in our model. Small differences between firms' intrinsic earnings can be magnified by expected industry's publicity into large differences in market prices, leading to large underpricing. In contrast, previous signalling models assume that the market price of shares is exogenous in equilibrium once the firm's type is known, and so they cannot generate underpricing when the difference between firms' intrinsic earnings is small.

Despite the obvious role of the industry's publicity, it is important to recall that the benefit of the industry's publicity to high-quality firms is an externality and hence such benefit reduces, rather than promotes, underpricing (Proposition 2.2). It is the informational cost generated by the industry's publicity that forces high-quality firms to underprice. As explained above, the informational cost increases with the industry's publicity because low-quality firms' temptation to mimic increases with the industry's publicity, which makes separation more difficult.

Our model explains why underpriced IPOs cluster in particular time and particular industry. The clustering of underpriced IPOs is a temporary phenomenon in new industries like the Internet industry, where publicity is likely to yield a large benefit initially. As the industry becomes

[15] Of course, a zero offer price is unrealistic. In subsection 5.1 we extend the model to generate a positive offer price in the underpricing equilibrium.

established, forecasts about earnings are more reliable and less susceptible to the influence of the industry's publicity. Moreover, as the industry matures, competition among firms in the same industry becomes more important than against firms in traditional sectors. In this case one firm's underpricing may hurt rather than benefit other firms in the same industry, and so IPOs with large underpricing are less likely to cluster.[16]

Our model also implies that clustering may vary over business cycles. The frequency and the magnitude of clustering are likely to be higher in economic expansions than in downturns, and in an easy credit market than in a tight credit market (other things being equal between the two markets). To see this, note that the cost of alternative funds, captured by parameter b, is likely to be lower in economic expansions than in downturns, and in an easy credit market than in a tight credit market. Because a reduction in b reduces the critical level $\underline{\rho}$ in (4.3), the condition $\rho > \underline{\rho}$ required for clustering is more easily satisfied. The explanation is as follows. When the cost of alternative funds is low, it is less costly to underprice IPO, because a firm can easily find alternative funds to make up for the shortfall in the IPO revenue. As a result, low-quality firms are more tempting to mimic high-quality firms. To separate successfully, a high-quality firm is more likely to resort to highly costly actions, such as underpricing IPO. For any given ρ, the amount of underpricing is also higher when b is lower.

5. Extensions and Robustness

In this section we extend the model to illustrate the robustness of the results and improve the model's quantitative implications. For example, by imposing a lower bound on equity financing, we show that the offer price can be positive in the underpricing equilibrium.

5.1. A Lower Bound on the IPO Revenue

A firm may face a constraint on how much non-equity fund it can obtain, and so it may be forced to obtain a minimum IPO revenue. Let this minimum be $Q_b s/p$, where $Q_b \in (0,1)$ is a

[16]This case corresponds to $\rho < 0$. Since underpricing is the best response to other firms' underpricing only if $\rho D > b - R_0(x_H - 1)$ (see (4.1) and (4.2)), under Assumption $1C$ there cannot be an underpricing equilibrium when $\rho < 0$.

constant. This specification incorporates the idea that alternative financiers are more willing to supply funds to a firm whose IPO has a larger price gain. Substituting the market price of shares in a separating equilibrium, we can rewrite the constraint $q \geq Q_b s/p$ for a high-quality firm as:

$$f \geq \frac{Q_b}{R_H - (1 + bx_H^{-1})(1 - q)} \equiv LB(q). \tag{5.1}$$

With this constraint, the separating action depicted by point A in Figures 1a and 1b may no longer be feasible to a high-quality firm. A scenario is depicted in Figure 4 for the case $Q_1 > 0$, in which the best separating action is given by point B. Because point B lies above the full-price curve $f = S_H(q)$, the IPO is underpriced. Such an underpricing equilibrium exists if and only if the curve $f = LB(q)$ crosses the curve $f = IND_L(q)$ before crossing $f = S_H(q)$. Equivalently, this requires $IND_L(Q_b) > S_H(Q_b)$, which is satisfied when Q_b is sufficiently close to 1.

Figure 4 here.

Two properties of the separating equilibrium here are in contrast with the simple model. First, an underpricing firm's offer price can be positive, as point B in Figure 4 illustrates. Second, the number of shares issued in IPO does not necessarily increase with earnings in the underpricing equilibrium. When R_H increases in Figure 4, for example, the curve $f = IND_L(q)$ shifts up but the curve $f = LB(q)$ shifts down. These two forces change f in opposite ways, and so the effect of R_H on f is ambiguous analytically. When the externality is sufficiently strong, however, f is likely to increase with R_H and firms underprice greatly, as in the simple model.

5.2. A Firm's Own Influence on Publicity

A firm may directly benefit from its underpricing, in addition to the industry's publicity. To allow for this benefit, let us return to the simple model and modify

$$R_H = R_0 x_H + \rho(\gamma d + D), \tag{5.2}$$

where d is the firm's own underpricing and $\gamma > 0$ is the relative impact of the firm's own underpricing on its expected earnings. The simple case before corresponds to $\gamma = 0$.

Now a firm cannot take R_H as given because its own decision directly affects R_H. Denote the part that the firm takes as given as $W \equiv R_0 x_H + \rho D$. Using (5.2) to compute R_I and substituting into (2.7), we obtain the market price of the firm's shares under the belief I as

$$p_I = \frac{1}{1 - I\rho\gamma}\left[IW + (1-I)R_0 - I\rho\gamma q/f - (1 + bE_I x^{-1})(1-q)\right]. \tag{5.3}$$

Restrict $0 \le \rho < 1/\gamma$ to ensure $p_I > 0$. The constraint $s \le p_H$ can be written as

$$f \ge q \Big/ \left[W - (1 + bx_H^{-1})(1-q)\right]. \tag{5.4}$$

Proposition 5.1. *There exist $\gamma_1 > 0$ and $\rho_1 \in (0, 1/(1+\gamma))$ such that an underpricing equilibrium exists if $\rho \in (\rho_1, 1/(1+\gamma))$ and $\gamma \le \gamma_1$. There exist $\gamma_2 > 0$ and $\rho_2 \in (0, 1/\gamma)$ such that a no-underpricing market equilibrium exists if $0 \le \rho < \rho_2$ and $\gamma \le \gamma_2$. Moreover, $\rho_1 < \rho_2$ and so the two market equilibria coexist when $\rho \in (\rho_1, \rho_2)$ and $\gamma \le \min\{\gamma_1, \gamma_2\}$.*

The proof of this proposition is omitted (see Cao and Shi 1999 or enclosed Appendix D). This proposition shows that the qualitative results here are similar to those in the simple model. In particular, IPO underpricing can cluster when the industry's publicity has a strong effect on the industry's product demand. This may be the case even when $\gamma > 1$, i.e., when a firm benefits more from its own publicity than from the industry's publicity.

5.3. Sequential Decisions

In the simple model we have assumed that different firms go to the IPO market at the same time. By this we do not mean that firms in reality literally make their IPO decisions at the same date but rather that some firms' IPO dates are close to each other so that one firm cannot change the IPO decision to take into account of observed actions by other firms. Although this interpretation is appealing, one may still want to know what happens if firms can modify their IPO decisions upon observing other firms' actions. We analyze this sequential game now and show that firms still tend to cluster their IPO underpricing decisions.

Consider only two firms, firm 1 and firm 2. Firm 1 goes to the IPO market at date 1 and firm 2 at date 2. To simplify matters, we assume that both firms have earnings only at date 2 and there is no time discounting. Let d_i be the amount of underpricing by firm $i = 1, 2$. Assume that, if firm i is perceived as a high-quality firm, expected earnings are given by R_H in (2.4), with D being replaced by $d_{i'}$ ($i' \neq i$). This specification keeps the gist of the interaction between firms in (2.4) and simplifies the algebra in this two-firm setup.

Given d_1, firm 2's pricing decision is analogous to that analyzed in the simple model. That is, if firm 2 is a low-quality firm, then $d_2 = 0$; if firm 2 is a high-quality firm, then

$$d_2 = \begin{cases} 0, & \text{if } d_1 < D_0 \\ \rho d_1 + R_0 x_H - 1 - b x_H^{-1}, & \text{if } d_1 \geq D_0, \end{cases}$$

where D_0 is defined in (4.1). Note that firm 2 responds to firm 1's underpricing positively.

Firm 1 anticipates this influence of its IPO pricing decision on firm 2's. Given firm 1's prior on firm 2's quality, the expected amount of firm 2's IPO underpricing is

$$\alpha \chi_{(d_1 > D_0)} (\rho d_1 + R_0 x_H - 1 - b x_H^{-1}),$$

where $\chi_{(d_1 > D_0)} = 1$ if $d_1 > D_0$ and 0 otherwise. Suppose firm 1 chooses $d_1 < D_0$. Then $d_2 = 0$ and there is no publicity from which firm 1 can benefit. In this case firm 1's best decision is $d_1 = 0$ and the payoff to both firms is identical to that in the no-underpricing equilibrium in the simple model. This can be a market equilibrium in the current case if and only if the payoff to firm 1 is not lower than that generated by the action $d_1 \geq D_0$.

Now suppose firm 1 chooses $d_1 \geq D_0$. If the market believes that the firm is of high-quality with probability I, the expected earning of the firm is

$$R_I = (1 - I) R_0 + I \left[R_0 x_H + \rho\alpha(\rho d_1 + R_0 x_H - 1 - b x_H^{-1}) \right].$$

Slightly change the earlier notation to denote $W = (1 + \rho\alpha) R_0 x_H - \rho\alpha(1 + b x_H^{-1})$. The market price of such a firm under the belief I is

$$p_I = \frac{1}{1 - I\alpha\rho^2} \left[IW + (1 - I) R_0 - I\alpha\rho^2 q/f - (1 + b E_I x^{-1})(1 - q) \right].$$

This is similar in form to the market price in the last subsection, with $\alpha\rho^2$ replacing $\rho\gamma$, and so firm 1's decision on the offer price can be analyzed analogously. To ensure a positive share price, we restrict $0 \leq \rho < \alpha^{-1/2}$. The proof of the following proposition is omitted (see Cao and Shi 1999 or enclosed Appendix E):

Proposition 5.2. *There exist $\rho_3, \rho_4 \in (0, \alpha^{-1/2})$ such that firm 1 underprices IPO if and only if it is of high-quality and $\rho \in (\rho_3, \rho_4)$. Firm 2 underprices IPO if and only if it is of high-quality and if firm 1 underprices IPO.*

Example 5.3. *Let $\alpha = 0.1$, $b = 0.2$, $x_L = 1$, $x_H = 1.18$ and $R_0 = 1.02$. Then, $\rho_3 = 1.75 < \rho_4 = 1.91$. Thus, the interval (ρ_3, ρ_4) can be non-empty.*

As the example shows, both firms underprice IPOs in some cases. More importantly, when firm 1 underprices IPO, firm 2 will do so as well if it is of high quality. Since such underpricing would not occur if there were only one firm in the industry or if publicity had no effect on the industry's expected demand, the result shows that the interaction between firms through expectations is important for the clustering of underpriced IPOs, just as in the case of simultaneous decisions. It is not surprising then that the underpricing equilibrium here also requires the externality to be strong (i.e., $\rho > \rho_3$).[17] In contrast to the case of simultaneous moves, however, too strong an externality (i.e., $\rho > \rho_4$) will destroy the underpricing equilibrium in the current case. This is because underpricing is costly and, when the externality is very strong, the amount of underpricing is too large to be desirable for firm 1 as the first mover in the game.

Multiplicity of equilibria disappears with sequential moves. However, this is an artifact of the exogenously fixed order of moves by the two firms. Being a first mover is costly in the current setup, because it must underprice sufficiently in order to entice the other firm to underprice. If firms can choose when to go to the IPO market, they have incentive to go to the market at dates that are very close to each other in order to explore the great externality. Then, the multiplicity analyzed in the simple model would reappear.[18]

[17] Firm 1's underpricing is not always echoed by firm 2, since firm 2 may turn out to be a low-quality firm. This uncertainty is eliminated in the case of simultaneous moves with the assumption of a large number of firms. As a result, the amount of underpricing is larger there than here.

[18] Tambanis and Bernhardt (1999) explicitly model the possibility that firms can delay the timing of their equity issue. However, they do not analyze IPO underpricing.

6. Conclusion

When firms signal their quality in initial public offerings of shares, an industry-wide uncertainty in product demand can induce many firms in the industry to underprice their IPOs at the same time. This clustering is a self-fulfilling phenomenon, which arises because the uncertainty makes expectations of the industry's product demand susceptible to the industry's publicity created by IPO performances. When other firms are expected to underprice, the industry's publicity is great and low-quality firms' temptation to mimic is strong, in which case a high-quality firm must underprice in order to separate itself out. When other firms are expected to not underprice, however, the industry's publicity is low and low-quality firms' temptation to mimic is weak, in which case a high-quality firm can signal its quality successfully by reducing the number of shares in IPO instead of underpricing.

Three aspects of the model are important for the clustering. The first is private information. If a firm's quality is public information, instead, the firm will free-ride on the industry's publicity, which eliminates underpricing altogether. The second is the feature that expected earnings of a high-quality firm respond more positively to product demand in the industry than those of a low-quality firm. This result we derive endogenously in a quality-ladder setup. The third is expectations. Whenever there is an underpricing equilibrium, there is also another equilibrium without underpricing. Thus, the clustering is not inevitable. No matter how strongly the industry's publicity affects expected product demand in the industry, it is optimal for a firm not to underprice IPO if it expects that other firms will not underprice.

Our emphasis on the clustering is a marked shift from the literature's emphasis on a single firm's underpricing. The analysis explains three features of a hot-issue market, especially the Internet craze in 1999. First, the clustering of large IPO underpricing is an industry-wide phenomenon. It occurs more often in industries that are uncertain in product demand, susceptible to the influence of publicity, and with severe private information regarding firms' qualities. As the industry matures, the clustering will become rare because forecasts about earnings become reliable and less susceptible to the influence of the industry's publicity. Second, the clustering of underpriced IPOs is fragile and short-lived. Even adverse news about a single firm can greatly affect all IPO performances in that industry, by inducing investors to switch the expectations from the underpricing equilibrium to the no-underpricing equilibrium.[19] In light of these two features, both the "hot-issue" market in Internet IPOs in 1999 and the subsequent cooling-off are outcomes of rational expectations about the new industry's performance. Finally, underpriced IPOs are more likely to cluster when the marginal cost of alternative funds is low, and so a large number of price gains in new shares occur more often in economic upturns than in downturns, and in an easy credit market than in a tight market. Thus, if the monetary authority wants to reduce the exuberance in the IPO market, it can do so by a tight monetary policy that reduces the loanable funds in the market.

[19] An example is the Biotech industry that experienced large underpricing in IPOs at the beginning of the 1990s. The heat over biotech stocks cooled down considerably when the Food and Drug Administration rejected several promising drugs such as Centocor Inc.'s Centoxin, a medicine meant to fight a deadly bacteria infection common in surgery patients.

Appendix

A. Proof of Lemma 2.1

Consider first a firm that produces a low-quality variety $i \in [\alpha, 1]$. Substituting $l_L = c(i)/(\beta c_0)$ and $c(i) = Y/\pi(i)$, we can rewrite this firm's net profit as $Y - Y/[\beta c_0 \pi(i)] - k_L$. Because competition drives net profit to 0 for this firm, the price of a low-quality variety is

$$\pi(i) = \pi^* \equiv \frac{1}{\beta c_0 (1 - k_L/Y)}.$$

With $c(i) = Y/\pi(i)$ and $l_L = c(i)/(\beta c_0)$, we obtain $c(i) = \beta c_0(Y - k_L)$ and $l_L = Y - k_L$. The earnings (after subtracting the labor cost), r_L, is equal to k_L, as stated in (2.2).

Now consider a firm that produces a high-quality variety $i \in [0, \alpha)$. Although a high-quality firm is the only producer for the specific variety, the firm must set price to prevent low-quality imitators from entering. If a firm enters and produces i with low-quality, the price of the good that this low-quality imitator will set is π^*, determined above. To prevent entry by low-quality imitators, a high-quality produce can set price at $\pi(i) = \pi^* - \varepsilon$, where $\varepsilon > 0$, which yields negative profit to an imitator. This pricing strategy succeeds for any $\varepsilon > 0$ and the optimal strategy is to set ε arbitrarily close to 0. Thus, in the limit, a high-quality variety's price is also π^*.

With the price π^*, a high-quality producer's labor input is $l_H = c(i)/c_0 = Y/(c_0\pi^*) = \beta(Y - k_L)$. Substituting this result into the definition of earnings r_H, it becomes clear that $r_H = (1-\beta)Y + \beta k_L$, as in (2.3). QED

B. Proof of Lemma 3.1

We establish the following lemma first:

Lemma B.1. $V^0(x_H), V^0(1) \leq R_\alpha - 1$.

Proof. To prove this lemma, we rewrite (3.1) and (3.2). Since $q_0/f_0 \leq p_\alpha$, the restriction $f_0 \in [0, 1]$ is equivalent to $p_\alpha \geq q_0 \geq 0$. With the price expression in (3.2), (3.1) becomes:

$$1 \geq q_0 \geq Q_0 \equiv \max\left\{0, 1 - \frac{R_\alpha - 1}{bE_\alpha x^{-1}}\right\}. \tag{B.1}$$

Since $R_\alpha > R_0 > 1$, $Q_0 < 1$. With $q_0 \geq Q_0$, (3.2) can be replaced by (3.10). Then, we have:

$$\begin{aligned}
V^0(x_H) &= (1-f_0)\left[R_\alpha - \left(1+bx_H^{-1}\right)(1-q_0)\right] \\
&\leq \{1-q_0/\left[R_\alpha - \left(1+bE_\alpha x^{-1}\right)(1-q_0)\right]\}\left[R_\alpha - \left(1+bx_H^{-1}\right)(1-q_0)\right] \\
&\leq \left\{1-q_0/\left[R_\alpha - \left(1+bx_H^{-1}\right)(1-q_0)\right]\right\}\left[R_\alpha - \left(1+bx_H^{-1}\right)(1-q_0)\right] \\
&= R_\alpha - \left(1+bx_H^{-1}\right)(1-q_0) - q_0 \ \leq R_\alpha - 1.
\end{aligned}$$

The first inequality follows from substituting the lower bound on f_0 in (3.10); the second inequality follows because the preceding expression is increasing in x; and the last inequality follows because the preceding expression is increasing in q_0. Similarly, $V^0(1) \leq R_\alpha - 1$. QED

For Lemma 3.1, we can verify the monotone and concavity features of $S_H(q)$ and $IND_L(q)$ directly. To prove the other properties in the lemma, note that

$$S_H(1) = 1/R_H < 1/R_\alpha < 1 - (R_\alpha - 1)/R_H < 1 - V^0(1)/R_H = IND_L(1).$$

The third inequality follows from Lemma B.1.

Consider first the case $Q_1 < 0$ (see Figure 1a). In this case the relevant range of q is $q \in [0, 1]$. Since $Q_1 < 0$, we have

$$S_H(0) = 0 < 1 - V^0(1)/(R_H - 1 - b) = IND_L(0).$$

Because $S_H(1) < IND_L(1)$, as shown above, $IND_L(q) > S_H(q)$ for both $q = 0$ and 1. To show $S_H(q) < IND_L(q)$ for all $q \in [0, 1]$, it suffices to show that $IND_L(q)$ crosses $S_H(q)$ from below if they ever cross each other in the positive quadrant. To show this crossing property, suppose that the two curves cross each other at $q_c \in [0, 1]$, i.e.,

$$1 - V^0(1)/\left[R_H - (1+b)(1-q_c)\right] = q_c \Big/ \left[R_H - (1+bx_H^{-1})(1-q_c)\right]. \qquad \text{(B.2)}$$

Computing the derivatives $IND_L'(q)$ and $S_H'(q)$ and substituting $V^0(1)$ from (B.2), we can show that $[IND_L'(q_c) - S_H'(q_c)]$ has the same sign as the following expression:

$$\begin{gathered}
[R_H - (1+b)(1-q_c)]q_c b x_H^{-1} + \left[R_H - (1+bx_H^{-1})(1-q_c) - q_c\right] \times \\
\left\{(1+b)\left[R_H - (1+bx_H^{-1})(1-q_c)\right] - [R_H - (1+b)(1-q_c)]\right\}.
\end{gathered}$$

The expression in $\{.\}$ is clearly positive. Also, Assumption $1A$ implies

$$R_H - (1+bx_H^{-1})(1-q_c) - q_c > R_H - (1+bx_H^{-1}) > 0.$$

Since $Q_1 < 0$, then $R_H - (1+b)(1-q_c) > V^0(1) > 0$. So, $IND'_L(q_c) > S'_H(q_c)$, as desired.

Consider now the case $Q_1 > 0$. Since $IND_L(q) < 0$ and $S_H(q) > 0$ if $0 \leq q < Q_1$, the two curves cannot cross each other in this range. Thus, consider only the range $q \geq Q_1$. In this range the above proof for the crossing property between $IND_L(q)$ and $S_H(q)$ goes through. Moreover, $IND_L(Q_1) = 0 < S_H(Q_1)$. Therefore, there is a unique crossing between the two curves.

Along $f = IND_L(q)$, a high-quality firm's payoff is

$$[1 - IND_L(q)][R_H - (1+bx_H^{-1})(1-q)] = V^0(1) \cdot \frac{R_H - (1+bx_H^{-1})(1-q)}{R_H - (1+b)(1-q)},$$

which is a decreasing function of q. Along $f = S_H(q)$, a high-quality firm's payoff is

$$[1 - S_H(q)]\left[R_H - (1+bx_H^{-1})(1-q)\right] = R_H - (1+bx_H^{-1})(1-q) - q,$$

which is an increasing function of q. QED

C. Proofs of Lemma 3.2 and Proposition 3.3

We prove Lemma 3.2 first. When out-of-equilibrium beliefs satisfy the Cho-Kreps criterion, the deviation to (f_b, q_b) in (3.11) from a pooling action generates the following gain to a high-quality firm:

$$\begin{aligned}
& (1-f_b)\left[R_H - (1+bx_H^{-1})(1-q_b)\right] - V^0(x_H) \\
= \; & (1-f_b)\left[R_H - (1+bx_H^{-1})(1-q_b)\right] - (1-f_b)\left[R_H - (1+b)(1-q_b)\right] \\
& + \{(1-f_b)\left[R_H - (1+b)(1-q_b)\right] - V^0(1)\} + \left[V^0(1) - V^0(x_H)\right] \\
= \; & b(1-x_H^{-1})(1-f_b)(1-q_b) - b(1-x_H^{-1})(1-f_0)(1-q_0) \\
= \; & \tfrac{b(1-x_H^{-1})}{1+b}\left[(1-f_b)R_H - (1-f_0)R_\alpha\right].
\end{aligned}$$

The first equality follows from adding and subtracting the same terms; the second equality follows from the fact that the term in $\{.\}$ is zero by the definitions of (f_b, q_b); the third equality follows from substituting the definitions of q_b and q_0. Then Lemma 3.2 is evident.

For Proposition 3.3, we locate the position of a pooling action (f_0, q_0). Since a pooling action must satisfy (3.10), it must lie on or above the curve $f = S_\alpha(q)$. Also, we can verify that

$IND_L(q_0) > f_0$ and so the point (f_0, q_0) must lie below the curve $f = IND(q)$. This implies $f_0 > f_b$ in the case $Q_1 > 0$ (see Figure 1*b*).

Consider first the case $Q_1 > 0$ (Figure 1*b*). Since $f_b < f_0$ in this case and $R_H > R_\alpha$, the gain to a high-quality firm from the deviation to (f_b, q_b) is strictly positive. Thus there cannot be a pooling equilibrium in this case. The only equilibrium is a separating equilibrium (f^*, q^*) defined by (3.13). The condition corresponding to this case, $Q_1 > 0$, becomes $R_H - R_0 < b$.

Now consider the case $Q_1 \leq 0$, where the separating actions are given by (3.12). Since

$$\begin{aligned} & (1-f_b)R_H - (1-f_0)R_\alpha = \tfrac{V^0(1)}{R_H-(1+b)}R_H - (1-f_0)R_\alpha \\ = \; & \tfrac{1-f_0}{R_H-(1+b)}\left[R_\alpha - R_H(1-q_0)\right], \end{aligned}$$

the gain to a high-quality firm from deviating from the pooling action to (f_b, q_b) is strictly positive if and only if $q_0 > 1 - R_\alpha/R_H$. Thus, (f^*, q^*) form a unique separating equilibrium against pooling actions with q_0 sufficiently close to 1. In this case the corresponding condition ($Q_1 \leq 0$) becomes $R_H - R_0 \geq b$.

The equilibrium established above is unique in the class of separating equilibria. To show that it is unique among all equilibria, we need to rule out pooling equilibria. A pooling action satisfies the Cho-Kreps intuitive criterion if and only if (3.3), (B.1), (3.10), $Q_1 \leq 0$ and $q_0 \leq 1 - R_\alpha/R_H$ are all satisfied. From the definition of Q_0 in (B.1), we have $Q_0 \leq 0$ if and only if $R_\alpha - 1 - bE_\alpha x^{-1} \geq 0$, i.e., iff

$$\alpha \geq \alpha_0 \equiv \frac{1+b}{R_H + b(1-x_H^{-1})}.$$

Note that $\alpha_0 \in (0,1)$ under Assumption 1*A*. Suppose that $\alpha \geq \alpha_0$ and so $Q_0 \leq 0$, in which case all $q_0 \in (0, 1 - R_\alpha/R_H]$ satisfy (B.1). For any such q_0, let f_0 solve (3.10) as an equality and note $f_0 \in (0,1)$. The payoff to a low-quality firm from this pooling action is

$$\left[1 - \frac{q_0}{R_\alpha - (1 + bE_\alpha x^{-1})(1-q_0)}\right]\left[R_\alpha - (1+b)(1-q_0)\right].$$

Both terms of the product are increasing functions of q_0 (for $q_0 > 0 > Q_0$). Thus the payoff is maximized by setting $q_0 = 1 - R_0/R_H$. For a pooling equilibrium to exist, this maximum pooling payoff must satisfy (3.3) with strict inequality. After substituting $R_\alpha = R_0 + \alpha(R_H - R_0)$ and

$E_\alpha x^{-1} = 1-\alpha(1-x_H^{-1})$, we can verify that the maximum pooling payoff satisfies (3.3) with strict inequality iff

$$\alpha - \frac{1-\alpha}{R_H - 1 - b + b\alpha(1-x_H^{-1})} + \frac{R_H - R_0(1+b)}{(R_H - R_0)(R_H - 1 - b)} > 0.$$

The left-hand side of the above inequality is an increasing function of α. When $\alpha = 0$, its value is negative. When $\alpha = 1$, its value has the same sign as

$$(R_H - R_0)(R_H - 1 - b) + R_H - R_0(1+b).$$

This is positive, since $R_H \geq R_0 + b$ (as $Q_1 \leq 0$) and the above expression has a value 0 when $R_H = R_0 + b$. Therefore there exists $\underline{\alpha} \in (0,1)$ such that (3.3) is satisfied with strict inequality for the above described (f_0, q_0) if $\alpha > \underline{\alpha}$. If $\alpha < \underline{\alpha}$, no pooling action satisfies the Cho-Kreps criterion, in which case the separating equilibrium is the unique equilibrium. QED.

References

[1] Allen, F. and G.R. Faulhaber, 1989, "Signalling by underpricing in the IPO market," *Journal of Financial Economics* 23, 303-323.

[2] Banerjee, A.V., 1992, "A simple model of herd behavior," *Quarterly Journal of Economics* 107, 797-817.

[3] Beatty, R.P. and J.R. Ritter, 1986, "Investment banking, reputation, and the underpricing of initial public offerings," *Journal of Financial Economics* 15, 213-232.

[4] Benveniste, L.M. and P.A. Spindt, 1989, "How investment bankers determine the offering price and allocation of new issues," *Journal of Financial Economics* 24, 343-361.

[5] Bikhchandani, S., Hirshleifer, D. and I. Welch, 1992, "A theory of fads, fashion, custom, and cultural change as informational cascades," *Journal of Political Economy* 100, 992-1026.

[6] Cao, M. and S. Shi, 1999, "Publicity and clustering of IPO underpricing," unpublished manuscript (available at http://php.indiana.edu/~shshi), Indiana University.

[7] Cho, I.K. and D.M. Kreps, "Signalling games and stable equilibria," *Quarterly Journal of Economics* 102, 179-221.

[8] Diamond, D. and P. Dybvig, 1983, "Bank runs, deposit insurance and liquidity," *Journal of Political Economy* 91, 401-419.

[9] Fudenberg, D. and J. Tirole, 1993, *Game Theory*, The MIT Press, Cambridge, Massachusetts.

[10] Grinblatt, M. and C.Y. Hwang, 1989, "Signalling and the pricing of new issues," *Journal of Finance* 44, 393-420.

[11] Grossman, G. and E. Helpman, *Innovation and Growth in the Global Economy*, MIT Press, Cambridge, MA (1991).

[12] James, C. and P. Wier, 1990, "Borrowing relationships, intermediation, and the cost of issuing public securities," *Journal of Financial Economics* 28, 149-171.

[13] Loughran, T. and J.R. Ritter, 2000, "Why don't issuers get upset about leaving money on the table in IPOs?" manuscript, University of Florida.

[14] Mauer, D.C. and L.W. Senbet, 1992, "The effect of the secondary market on the pricing of initial public offerings: theory and evidence," *Journal of Financial and Quantitative Analysis* 27, 55-79.

[15] Michaely, R. and W.H. Shaw, 1994, "The pricing of initial public offerings: tests of adverse selection and signaling theories," *Review of Financial Studies* 7 (2), 279-319.

[16] Prial, D., 1999, "IPO delays, cancellations are on the rise," *Wall Street Journal* (April 19).

[17] Ritter, J., 1984, "The 'hot issue' market of 1980," *Journal of Business* 57, 215-241.

[18] Rock, K., 1986, "Why new issues are underpriced," *Journal of Financial Economics* 15, 187-212.

[19] Slovin, M.B. and J.E. Young, 1990, "Bank lending and initial public offerings," *Journal of Banking and Finance* 14, 729-740.

[20] Tambanis, D. and D. Bernhardt, 1999, "Adverse selection and the timing of equity issue," manuscript, University of Illinois at Urbana Champaign.

[21] Welch, I., 1989, "Seasoned offerings, imitation costs, and the underpricing of initial public offerings," *Journal of Finance* 44, 421-449.

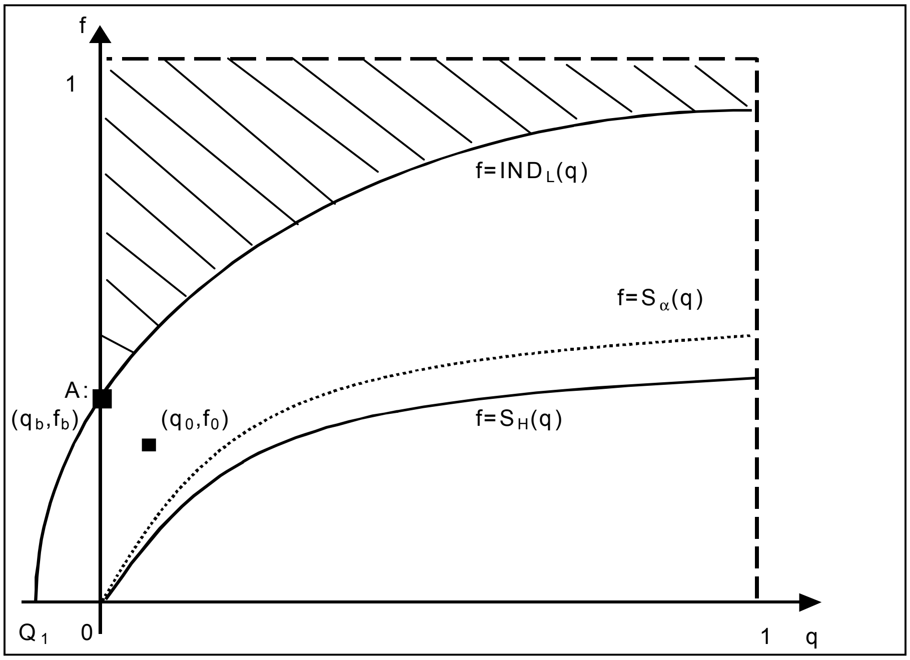

Figure 1a Deviations by a high-quality firm: $Q_1 < 0$

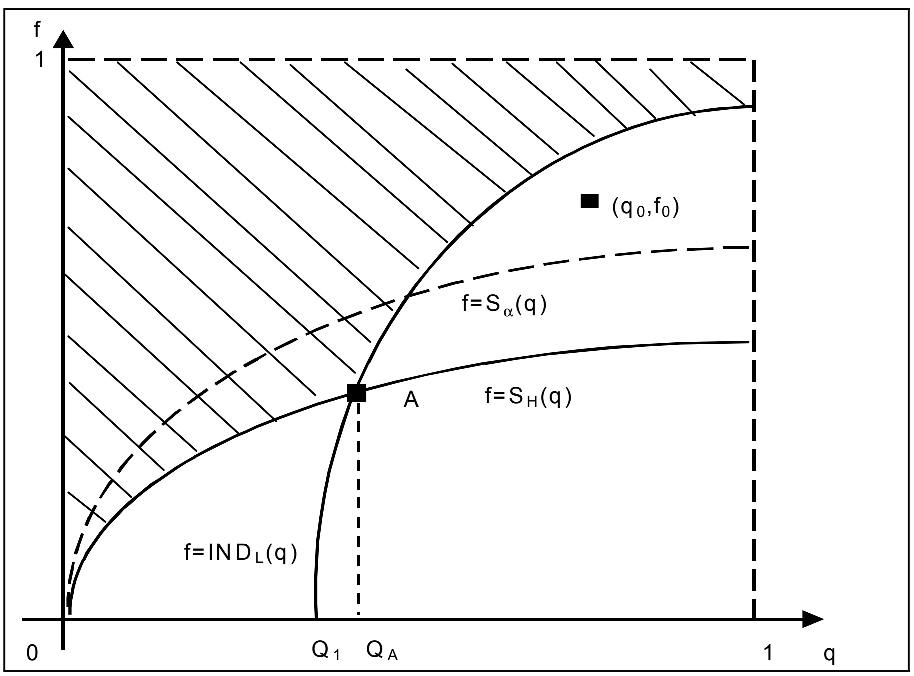

Figure 1b Deviations by a high-quality firm: $Q_1 > 0$

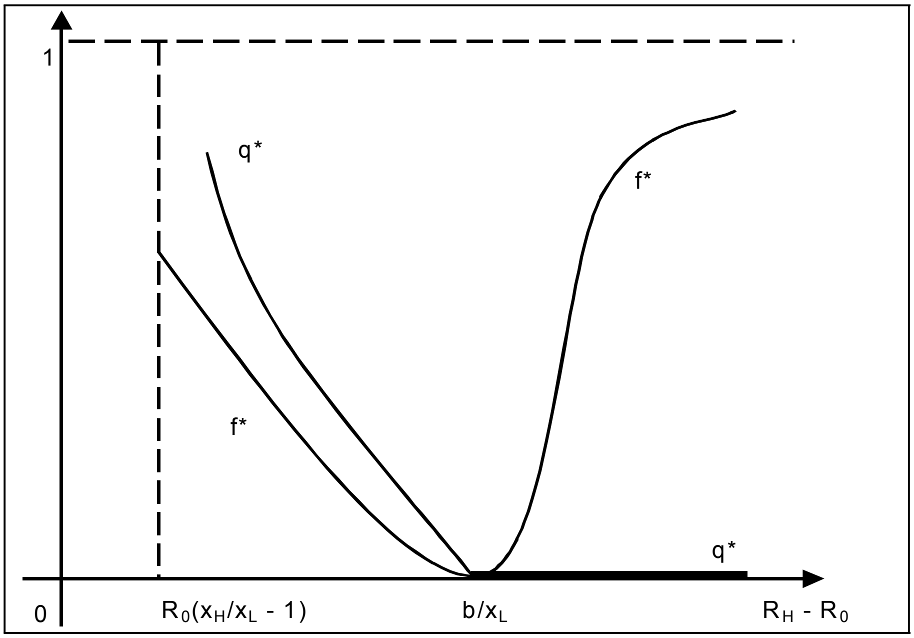

Figure 2 Dependence of (f, q) on the earnings difference between a high-quality and a low-quality firm in the separating equilibrium

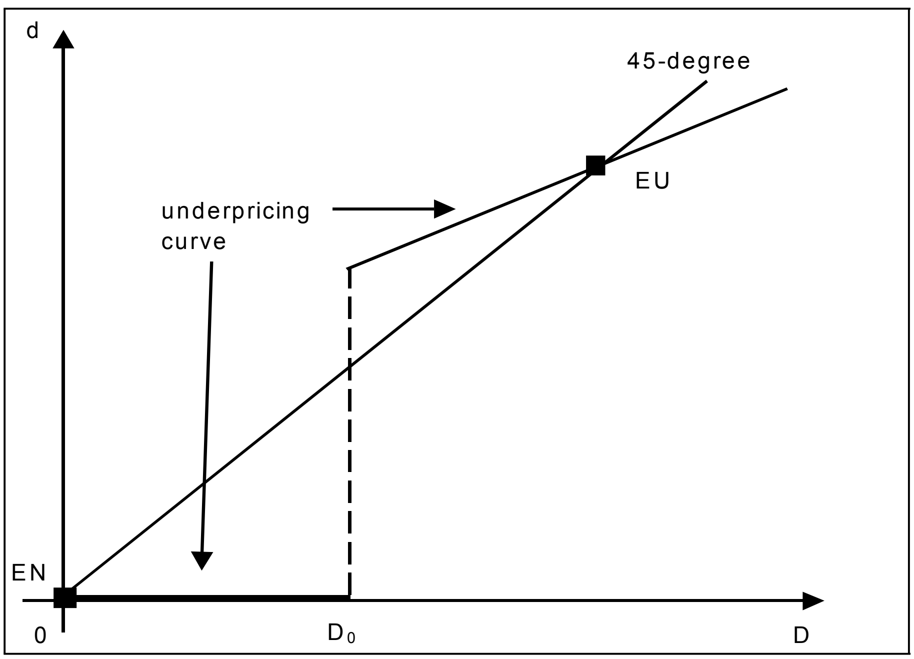

Figure 3 Market equilibria

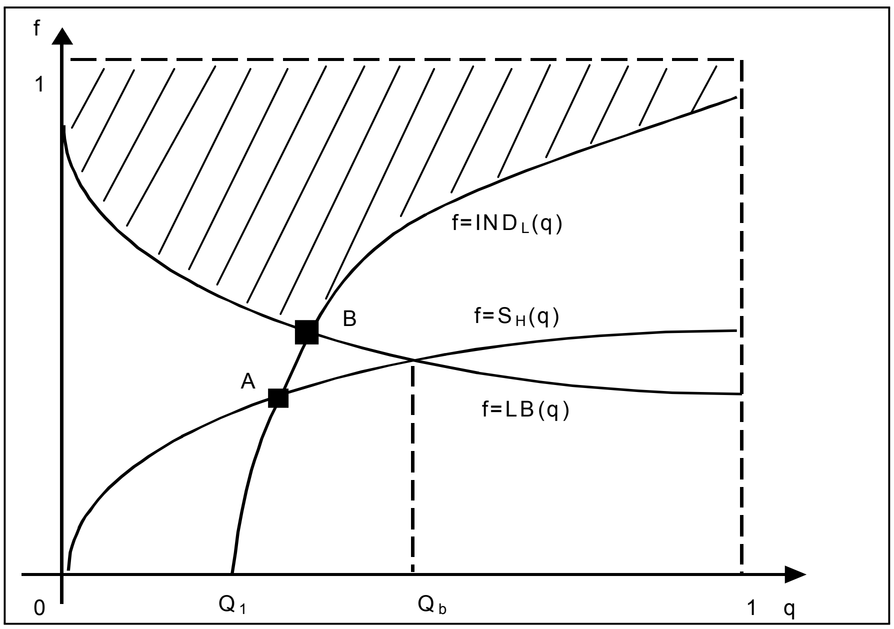

Figure 4 A separating equilibrium when there is a lower bound on the amount of equity financing

Supplementary Appendix for

"Signalling in the Internet Craze of Initial Public Offerings"

by Melanie Cao and Shouyong Shi

D. Proof of Proposition 5.1

Let V_L^0 be the payoff to a low-quality from a pooling action (f_0, q_0). As in the simple model, we find separating actions that generate lower payoffs to a low-quality firm than in a pooling equilibrium. Then we choose the best among these actions as a candidate for the action of a high-quality firm in a separating equilibrium. If a low-quality firm deviates from the pooling action to an action (f, q) and is perceived as a high-quality firm, the payoff is

$$(1-f)\left[W+\rho\gamma(p_H-s)-(1+b)(1-q)\right]=\frac{1-f}{1-\rho\gamma}\left[W-\rho\gamma q/f-(1+z)(1-q)\right],$$

where $W = R_0 x_H + \rho D$ and $z = \rho\gamma b/x_H + (1-\rho\gamma)b$. This payoff is less than that in the pooling equilibrium if and only if

$$\begin{array}{ll} q < G(f) \equiv \frac{1+z-W+(1-\rho\gamma)V_L^0/(1-f)}{1+z-\rho\gamma/f}, & \text{for } f > \frac{\rho\gamma}{1+z}; \\ q > G(f), & \text{for } f < \frac{\rho\gamma}{1+z}. \end{array}$$

Figures $5a$ and $5b$ here.

Let us divide the proof into two cases.

Case 1: $W > (1+z)[1+(1-\rho\gamma)V_L^0/(1+z-\rho\gamma)]$. This case is depicted in Figure $5a$. Let $S_H(q)$ now denote the right-hand side of (5.4) and let its inverse be S_H^{-1}. It can be shown that there exists $\gamma_1 > 0$ such that $G(f) > S_H^{-1}(f)$ in the region $f < \rho\gamma/(1+z)$ if $\gamma \le \gamma_1$, as depicted in Figure $5a$. Restrict attention to $\gamma \le \gamma_1$. In this case the relevant region is $f > \rho\gamma/(1+z)$ and the shaded area is the set of actions that yield lower payoff to a low-quality firm but may yield higher payoff to a high-quality firm than in the pooling equilibrium. We can verify the following properties for the segment of $G(f)$ with $f > \rho\gamma/(1+z)$:

(1a) $G(f) > 0$ iff $f > 1 - (1-\rho\gamma)V_L^0/(W-1-z)$ (i.e., iff f is higher than point A).

(1b) $G'(f) > 0$ for all $f > 1 - (1-\rho\gamma)V_L^0/(W-1-z)$.

(1c) The payoff to a high-quality firm from taking actions along $q = G(f)$ is decreasing in f.

These properties imply that, if $\gamma \leq \gamma_1$, the best deviation for a high-quality firm from a pooling equilibrium is point A in Figure 5a. In this case, $q = s = 0$ and there is underpricing as in the corresponding case in the simple model.

Case 2: $W < (1+z)[1 + (1-\rho\gamma)V_L^0/(1+z-\rho\gamma)]$. In this case, the best deviations for a high-quality firm in the region $f < \rho\gamma/(1+z)$ lie on the curve $f = S_H(q)$ and, by property (2c) below, they are strictly dominated by the action at point A in Figure 5b. Thus, it suffices to consider only the region $f > \rho\gamma/(1+z)$. The curve $q = G(f)$ for $f > \rho\gamma/(1+z)$ is depicted by Figure 5b, where the shaded area is the set of deviations that are feasible to a firm (when perceived as a high-quality firm as a result of deviation) and that generate lower payoffs to a low-quality firm than in the pooling equilibrium. A lengthy exercise can establish the following properties, some of which are depicted in Figure 5b:

(2a) There exists a level $f_c \in (\rho\gamma/(1+z), 1)$ such that the curve $q = G(f)$ is decreasing in f for $f \in (\rho\gamma/(1+z), f_c)$ and increasing in f for $f \in (f_c, 1)$.

(2b) $S_H(1) = 1/W > \rho\gamma/(1+z)$ and $G(1/W) < 1$. That is, the intersection between the curve $f = S_H(q)$ and $q = 1$ lies in the region $q > G(f)$ and $f > \rho\gamma/(1+z)$. Since the curve $f = S_H(q)$ starts outside this region when q is small, there is at least one intersection between $f = S_H(q)$ and $q = G(f)$, as depicted by point A in Figure 5b.

(2c) A high-quality firm's payoff from actions along the curve $f = S_H(q)$ increases in q.

(2d) A high-quality firm's payoff from actions along the curve $q = G(f)$ (for $f > \rho\gamma/(1+z)$) decreases in f for all $f \geq (\rho\gamma/W)^{1/2}$.

(2e) There exists $\gamma_2 > 0$ such that, if $\gamma \leq \gamma_2$, the intersection (point A) has $f \geq (\rho\gamma/W)^{1/2}$.

These properties imply that, if $\gamma \leq \gamma_2$, the payoff to a high-quality firm from deviating from the pooling action is maximized at the intersection between the curve $f = S_H(q)$ and $q = G(f)$, such as point A in Figure 5b. There is no underpricing in this case.

When α is sufficiently small, in both case 1 and case 2 one can also show that the payoff at point A to a high-quality firm is higher than the payoff in the pooling equilibrium, provided that the market views such deviation as coming from a high-quality firm. Thus, the action given by point A is the separating equilibrium that satisfies the Cho-Kreps criterion. Substituting $W = R_0 x_H + \rho D$ and noting that the payoff to a low-quality firm is $R_0 - 1$ in the absence of pooling (thus V_L^0 in the above analysis is replaced by $R_0 - 1$), we have,

$$\begin{aligned} d &= p_H = \tfrac{1}{1-\rho\gamma}\left[\rho D + R_0 x_H - 1 - b/x_H\right], \\ & \quad \text{if } R_0 x_H + \rho D > (1+z)\left[1 + \tfrac{(1-\rho\gamma)(R_0-1)}{1+z-\rho\gamma}\right], \end{aligned} \tag{D.1}$$

$$d = 0 \qquad \text{if } R_0 x_H + \rho D < (1+z)\left[1 + \frac{(1-\rho\gamma)(R_0 - 1)}{1+z-\rho\gamma}\right]. \tag{D.2}$$

To solve for market equilibria, impose symmetry $d = D$. Doing so for case 1 we get:

$$d = D = \frac{R_0 x_H - 1 - b/x_H}{1 - \rho(1+\gamma)}.$$

Thus, $d > 0$ only if $\rho < 1/(1+\gamma)$. Also, (D.1) must be satisfied in order to have $D > 0$, i.e.,

$$R_0 x_H + \rho \frac{R_0 x_H - 1 - b/x_H}{1 - \rho(1+\gamma)} > (1+z)\left[1 + \frac{(1-\rho\gamma)(R_0 - 1)}{1+z-\rho\gamma}\right]. \tag{D.3}$$

Note that z and $(1-\rho\gamma)/(1+z-\rho\gamma)$ are decreasing functions of ρ and so is the right-hand side of the above inequality. The left-hand side is an increasing function of ρ. Since the inequality is satisfied for $\rho = 1/(1+\gamma)$ and violated for $\rho \to 0$, there exists a critical level $\rho_1 \in (0, 1/(1+\gamma))$ such that the above inequality is satisfied if and only if $\rho > \rho_1$. Therefore, an underpricing equilibrium exists if $\rho_1 < \rho < 1/(1+\gamma)$ and $\gamma \leq \gamma_1$.

For the no-underpricing equilibrium, impose $d = D = 0$ in case 2. The equilibrium exists if

$$R_0 x_H < (1+z)\left[1 + \frac{(1-\rho\gamma)(R_0 - 1)}{1+z-\rho\gamma}\right]. \tag{D.4}$$

The right-hand side of this inequality is a decreasing function of ρ. The inequality is satisfied when $\rho \to 0$ and violated when $\rho \to 1/\gamma$. Thus, there exists $\rho_2 \in (0, 1/\gamma)$ such that the inequality is satisfied for $0 < \rho < \rho_2$. If $\gamma \leq \gamma_2$, in addition, the no-underpricing equilibrium exists.

Comparing (D.3) and (D.4) we can immediately show $\rho_1 < \rho_2$. Therefore, the underpricing equilibrium and the no-underpricing equilibrium coexist if $\rho \in (\rho_1, \rho_2)$ and $\gamma \le \min\{\gamma_1, \gamma_2\}$. This completes the proof of Proposition 5.1. QED

E. Proof of Proposition 5.2

We have already argued in the text that firm 2 underprices only if firm 1 underprices sufficiently (i.e., if $d_1 \ge D_0$). Analogous to the derivation of (D.1) in Appendix D, we have:

$$d_1 = \frac{1}{1-\alpha\rho^2}(W - 1 - b/x_H), \tag{E.1}$$

$$\text{if } W > (1+\alpha\rho^2 b x_H^{-1} + (1-\alpha\rho^2)b)\left[1 + \frac{(1-\alpha\rho^2)(R_0-1)}{1+\alpha\rho^2 b x_H^{-1} + (1-\alpha\rho^2)b - \alpha\rho^2}\right], \tag{E.2}$$

where $W = (1+\rho\alpha)R_0 x_H - \rho\alpha(1+bx_H^{-1})$. The underpricing equilibrium has $q = G(f) = 0$. With V_L^0 being set to $R_0 - 1$, $G(f) = 0$ implies:

$$f = 1 - \frac{(1-\alpha\rho^2)(R_0-1)}{W - \left[1+\alpha\rho^2 b x_H^{-1} + (1-\alpha\rho^2)b\right]}. \tag{E.3}$$

For firm 1 to underprice, d_2 must also be positive and so we need $d_1 \ge D_0$, i.e.

$$W - 1 - b/x_H \ge \frac{1-\alpha\rho^2}{\rho}\left[b - R_0\left(x_H - 1\right)\right]. \tag{E.4}$$

Note that W increases in ρ and the right-hand side of (E.2) decreases in ρ. Moreover, (E.2) is satisfied when $\rho \to \alpha^{-1/2}$ and is violated when $\rho \to 0$. Then, there exists $\rho_a \in (0, \alpha^{-1/2})$ such that (E.2) is satisfied if and only if $\rho \in (\rho_a, \alpha^{-1/2})$. Similarly, there exists $\rho_b \in (0, \alpha^{-1/2})$ such that (E.4) is satisfied if and only if $\rho \in [\rho_b, \alpha^{-1/2})$. Let $\rho_3 = \max\{\rho_a, \rho_b\}$. Then both (E.2) and (E.4) are satisfied if and only if $\rho \in (\rho_3, \alpha^{-1/2})$.

In addition to the requirement $\rho \in (\rho_3, \alpha^{-1/2})$, the payoff to firm 1 (when it is high-quality) must be higher with $d_1 > 0$ than with $d_1 = 0$ in order for the firm to underprice. With $d_1 = 0$, the payoff to high-quality firm 1 is

$$\begin{aligned} & (1-f^*)\left[R_0 x_H - \left(1+\tfrac{b}{x_H}\right)(1-q^*)\right] = R_0 x_H - \left(1+\tfrac{b}{x_H}\right)(1-q^*) - q^* \\ = \; & (R_0 - 1)\left[R_0 x_H - (1+b/x_H)(1-q^*)\right]/\left[R_0 x_H - (1+b)(1-q^*)\right] \end{aligned}$$

where the inequalities come from substituting the definitions of (f^*, q^*) in (3.13). When $d_1 > 0$ in (E.1), $q = 0$ and f is given by (E.3). The total return to shareholders is $(W - 1 - bx_H^{-1})/(1 - \alpha\rho^2)$ and the payoff to high-quality firm 1 from underpricing is

$$\frac{(R_0 - 1)(W - 1 - bx_H^{-1})}{W - \left[1 + \alpha\rho^2 bx_H^{-1} + (1 - \alpha\rho^2)b\right]}.$$

Substituting W and simplifying, we can show that the firm's payoff is higher with underpricing than without if and only if

$$\frac{1 - \alpha\rho^2}{1 + \alpha\rho} > \frac{(1 - q^*)(R_0 x_H - 1 - b/x_H)}{R_0 x_H - (1 + b/x_H)(1 - q^*)}.$$

There exists $\rho_4 \in (0, \alpha^{-1/2})$ such that the above condition is satisfied if and only if $0 \leq \rho < \rho_4$. The level ρ_4 is not necessarily greater than ρ_3. Only when $\rho_4 > \rho_3$ and $\rho \in (\rho_3, \rho_4)$ does high-quality firm 1 underprice IPO. QED

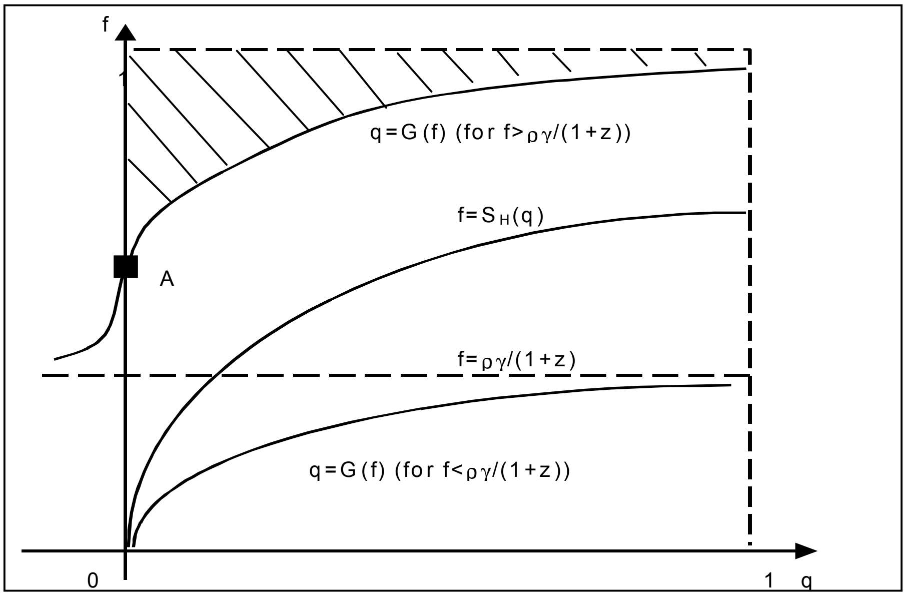

Figure 5a When a high-quality firm has its own influence on publicity: Case 1 (large W)

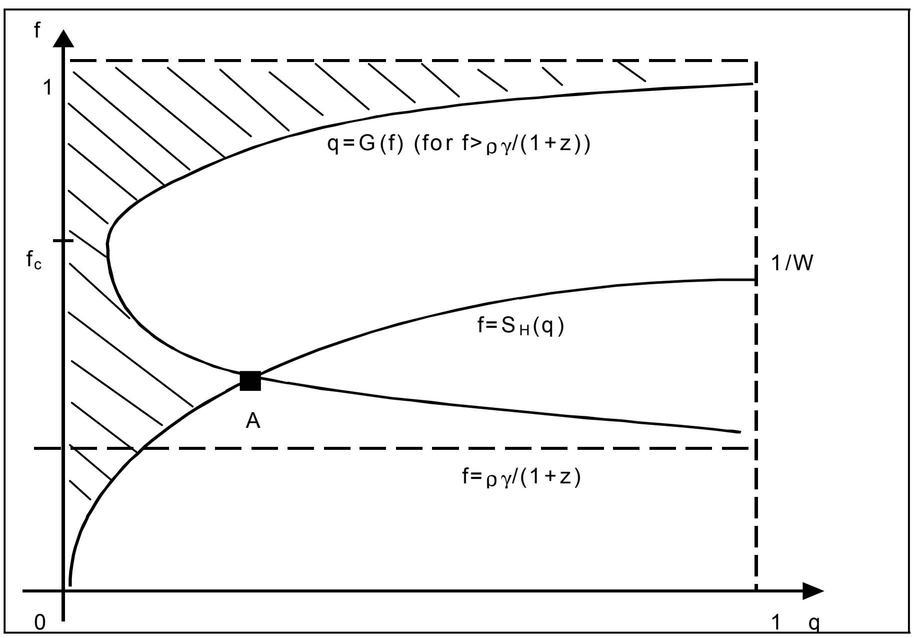

Figure 5b When a high-quality firm has its own influence on publicity: Case 2 (small W)