Investments of Company Stock in 401(k) Plans: Impacts of Bankruptcy Risk and Holding Restrictions *

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First verion: May 2004
Current verion: December 2004

Abstract

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How does bankruptcy risk affect employees' optimal asset allocation? How much is the employee's welfare loss from bearing the bankruptcy risk when required to invest in company stock? How much is the combined welfare loss due to both holding restrictions and bankruptcy risk? All of the questions are extremely important in today's world of pension management, yet none of them has been addressed in the literature to date. This study fills this important gap.

Using a utility maximization model with a CRRA preference, we find that holding restrictions and bankruptcy risk both have an important bearing on optimal allocation and welfare losses. The total welfare loss can be easily as high as 50% of the stock value. For a typical firm, the welfare loss due to bankruptcy risk amounts to 20% to 30% of the stock value.

Keywords*: Pension Investments, Restricted Stock, Bankruptcy Risk, Asset Allocation.*

JEL classifications: G11; G23; G33.

*We gratefully acknowledge the financial support from the *Social Sciences and Humanities Research Council of Canada*. Cao also acknowledges research support from the Schulich School of Business at York University. We thank Xifeng Diao, Moshe Milevsky and participants of the 2004 meeting of the Northern Finance Association for their useful comments.

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1. Introduction

According to Mitchell and Utkus (2002), there are more than 700,000 corporate defined-contribution pension plans in the U.S., covering about 55 million workers and managing over $2 trillion in assets. Among these 700,000 defined contribution plans, the majority of the companies have either encouraged or required their employees to invest in their own stocks, either through the 401(k) plans, or through tax-qualified profit-sharing plans and/or employee stock ownership plans. In fact, 401(k) plans account for more than 50% of the 700,000 plans, and over 80% of the assets (Mitchell and Utkus, 2002). Benartzi (2001) states that about a third of the assets in large retirement savings plans are invested in company stocks while about a quarter of the discretionary contributions are invested in company stocks. Moreover, employers' contributions toward company stocks seems to induce employees to invest even more in company stocks.[1]

Table 1 lists some well-known U.S. companies which contribute their own stocks to 401(k) plans. It is seen that the concentration on firm's own stock in pension investments is disturbingly high. For instance, in 2002, Procter & Gamble invested 92% of pension value in its own stock while McDonalds invested 74%. The exposure to company stock can bring good fortune to plan-holders if the company's stock does very well. However, it can bring large losses to employees when the company stock performs poorly, as apparent in Panel B of Table 1. To make matters worse, if the company goes bankrupt, the employees not only suffer tremendous financial losses in their pension investments, but also lose their jobs. Enron's collapse dramatically illustrated this point because a large percentage of Enron's retirement plan was invested in Enron shares. According to Enron's 401(k) plan, for every dollar of employees' contribution, Enron contributed fifty cents worth of Enron's stock and employees were not allowed to sell the stock until they reach age 50. Panel B of Table 1 shows that 41% of Enron's pension was invested in company stock and its stock value subsequently dropped by more than 99% between March 2000 and December 2001. Enron eventually declared bankruptcy on December 2, 2001. Many other companies (e.g.,

[1] Using a framework based on ambiguity aversion, Boyle Uppal and Wang (2004) propose a model to explain why it could be even optimal to hold company's stock in pension plans. Even and Macpherson (2003) empirically examine the causes and consequences of holding company stock in pension plans.

Polaroid, Lucent Technologies, and Northern Telecom) had suffered similar losses, although they avoided bankruptcy.

The immediate concern over investing in company stock is the under-diversification problem. Some researchers have discussed the cost of under-diversification. For example, Brennan and Torous (1999) show that the certainty equivalent of investing one dollar in a single stock for 10 years is only \$0.36 in a utility maximization framework with a constant relative risk aversion of 2. Meulbroek (2002) uses a well-diversified stock portfolio as a benchmark to assess the cost of holding company stock. With the empirically estimated parameters, she shows that an employee would sacrifice about 42% of the market value of the firm's stock simply due to lack of diversification.

There are other concerns in addition to the under-diversification problem. First, every firm is subject to the possibility of bankruptcy. How does bankruptcy risk affect employees' optimal asset allocation within their pension portfolios? How much is the employee's welfare loss due to bearing the bankruptcy risk when required to invest in company stock? Second, the holding restrictions undermine employees' ability to optimally structure their pension portfolios. How much is employees' welfare loss due to holding restrictions? How much is the combined welfare loss due to both holding restrictions and bankruptcy risk? Under what conditions are welfare losses significant?

This paper sets out to address the above questions. Some studies have examined the impact of holding restrictions and / or vesting requirements on the private valuation and incentive effects of company stocks and options. Examples include Kahl, Liu and Longstaff (2003), Cao and Wei (2004), Hodder and Jackwerth (2004), and Ingersoll (2004).[2] However, to our best knowledge, no one has ever investigated how bankruptcy risk affects optimal asset allocations and how much welfare loss it causes. With the precedence of Enron and the prevalent practice of placing a large portion of pension investments in company stocks, it is extremely important to rigorously examine the impact of bankruptcy risk and investigate how it interacts with holding restrictions for pension investments. Our study therefore fills an important gap in the literature.

We use a utility maximization framework to study the joint effect of holding restrictions and

[2] In the context of assessing the pros and cons of expensing executive stock options, Chance (2004) offers a thorough survey of the literature on the valuation of executive stock options.

bankruptcy risk. The employee is assumed to have a CRRA utility and is required to hold a fixed number of company shares for a given period of time. Aside from the vesting period, the employee's planning horizon also includes a free allocation period. She optimally allocates her liquid wealth between the risk-free bond and the market portfolio to maximize the expected utility of the terminal wealth. The company is subject to bankruptcy in which case the stock holding is worth zero. For simplicity, the bankruptcy process is assumed to be independent of the stock and market portfolio processes. Welfare losses are measured as the difference between the stock's market value and its certainty equivalent value.

Our study shows that holding restrictions and bankruptcy risk can affect optimal allocations and cause significant welfare losses. The total welfare loss can be easily as high as 50% of the stock value. The contribution of bankruptcy risk to the total welfare loss is substantial. For instance, for a vesting period of 10 years, calculations based on calibrated parameter values show that bankruptcy risk can lead to a discount of 20% to 30% of the stock value. Investing company's own stock in 401(k) plans is a costly strategy.

The remainder of the paper is organized as follows. Section 2 lays out the model and delineates the measures for welfare loss due to holding restrictions and bankruptcy risk. Section 3 presents numerical results and analyses. Section 4 contains a brief study of a sample of companies which contribute their own stocks to 401(k) plans. The last section summarizes and concludes the paper. Tables are relegated to the end of the paper.

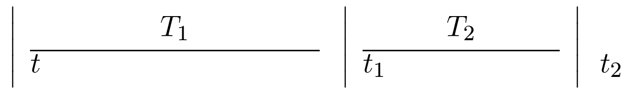
2. The Setup

2.1. Framework of Optimal Asset Allocation

Consider an employee whose risk preference is described by the constant-relative-risk-aversion (CRRA) utility function:

$$U(W) = \frac{W^{1-\gamma}}{1-\gamma}, \tag{2.1}$$

where $\gamma > 0$ is the coefficient of relative risk aversion. The employee works for a firm for a period of $(T_1 + T_2)$ years, and her objective is to maximize her expected utility at the end of the period. There are two distinct sub-periods as shown below.



At the beginning of the period, time t, the firm contributes to the employee's pension with the firm's shares, the current value of which is S. The vesting period of the contributed shares is T_1, meaning that the employee is not allowed to sell the shares until time t_1. After time t_1, the employee can freely allocate her wealth in the second period of T_2 years. We can think of time t_2 as the retirement date.[3]

The price of the company's stock follows a geometric Brownian motion:

$$\frac{dS}{S} = \mu_s dt + \sigma_s dz_s, \tag{2.2}$$

where μ_s and σ_s are the mean return and standard deviation. The employee is assumed to have an initial liquid wealth of $W_0 - S$ invested in the pension. Within the first T_1 years, she can freely allocate the liquid wealth between a risk-free bond and the market portfolio, the processes of which are,

$$\frac{dB}{B} = r_f dt, \tag{2.3}$$

and

$$\frac{dM}{M} = \mu_m dt + \sigma_m dz_m, \tag{2.4}$$

where r_f is the risk-free interest rate, μ_m and σ_m are the mean return and standard deviation of the market portfolio, and $E(dz_s dz_m) = \rho dt$. For base-case analysis, we utilize the continuous CAPM relationship between the returns of the stock and the market: $\mu_s = r_f + \frac{\sigma_s \rho}{\sigma_m}(\mu_m - r_f)$. In Section 3.2.2, we examine the case where the stock earns a non-zero abnormal return.[4]

[3] Please note that our setup also applies to situations where no explicit vesting requirements are in place. As shown by Benartzi (2001), Liang and Weisbenner (2002), and Brown, Liang and Weisbenner (2004), employees tend to voluntarily invest in their own company's stock for various reasons. In this paper, we are more concerned with the impacts of (as opposed to the reasons for) such allocation practices.

[4] In a bona fide, general equilibrium framework, the expected return of the stock should incorporate the bankruptcy

The firm is subject to bankruptcy with probability p per year. For simplicity, we assume the bankruptcy process to be independent of the stock's price process in (2.2). The status of the firm (i.e., whether it is bankrupt or not) is revealed at the end of each year.[5] Should the firm go into bankruptcy within the stock's vesting period, the employee's stock holding will be worth zero, and she will re-allocate her remaining wealth between the risk-free bond and the market portfolio. In this case, given the above assumptions, her holding of the market will be the Merton's myopic constant proportion, $\overline{x}_m = \frac{\mu_m - r_f}{\gamma \sigma_m^2}$ (Merton, 1969, 1971). If the firm remains healthy, then at the end of the vesting period t_1, the employee will sell the stock holding and re-allocate her total wealth according to the myopic rule.

The challenge is to find the employee's optimal allocation rule within the vesting period when the firm is healthy. Even without bankruptcy risk, this allocation rule is already time and state dependent as shown in Kahl, Liu and Longstaff (2003). Within our setup, even if we assume a constant allocation rule between two status revelations (e.g., a year), the number of distinct weights is still equal to the number of status revelations in the vesting period. The optimization is clearly not tractable even with numerical procedures. To overcome this difficulty, we make a further simplifying assumption: the employee will adhere to a fixed weight in the market portfolio until the firm goes bankrupt at time τ or the end of the vesting period t_1, whichever comes first. It should be noted that, just as the Merton's myopic allocation rule, the fixed weight in the market portfolio is not the same as a buy-and-hold strategy. Indeed, continuous rebalancing is required to maintain the fixed weight.

Denote x_m as the percentage of the employee's liquid wealth invested in the market portfolio. The employee optimally chooses x_m to maximize her utility of terminal wealth W at time t_2.

risk. In this paper, our focus is on the impacts of holding restrictions and bankruptcy risk, rather than the equilibrium determination of stock returns in the presence of bankruptcy risk. The CAPM relationship is assumed purely for convenience - to obtain the Merton result in the absence of holding restrictions. One can think that the CAPM equation together with the bankrutpcy process jointly determine the true return process of the stock.

[5]The revelation period of one year is assumed for ease of exposition only. In numerical analyses, we can allow this period to be any length. Moreover, the assumption of independence between bankruptcy and stock return processes is not strictly necessary. In fact, our numerical procedures can easily incorporate a correlation. Aside from simplicity, our assumption also mimicks the fact that corproate bankruptcy is not always related to stock performances, at least not until the true status of the firm is revealed. Enron and Worldcom are good examples.

Specifically, given the processes in (2.2), (2.3), and (2.4), the optimization problem becomes

$$\max_{x_m} E[U(W) = U^* \tag{2.5}$$

where

$$W = \begin{cases} W_1 = \left[(W_0 - S)e^{\mu_p T_1 + \, x_m \sigma_m \sqrt{T_1}\varepsilon_m^{t_1}} + Se^{(\mu_s - \frac{1}{2}\sigma_s^2)T_1 + \, \sigma_s \sqrt{T_1}\varepsilon_s^{t_1}}\right] e^{\mu_p' T_2 + \overline{x}_m \sigma_m \sqrt{T_2}\varepsilon_m^{t_2}} & \text{if } \tau > t_1, \\ W_2 = (W_0 - S)e^{\mu_p(\tau - t) \, + \, x_m \sigma_m \sqrt{\tau - t}\varepsilon_m^{\tau}} e^{\mu_p'(t_2 - \tau) \, + \, \overline{x}_m \sigma_m \sqrt{t_2 - \tau}\varepsilon_m^{t_2}} & \text{if } \tau \leq t_1, \end{cases}$$

$$\mu_p = r_f + x_m(\mu_m - r_f) - \tfrac{1}{2}x_m^2 \sigma_m^2,$$

$$\mu_p' = r_f + \overline{x}_m(\mu_m - r_f) - \tfrac{1}{2}\overline{x}_m^2 \sigma_m^2,$$

and $\varepsilon_m^{t_1}$, $\varepsilon_s^{t_1}$, ε_m^{τ}, and $\varepsilon_m^{t_2}$ are all standard normal variables among which only $\varepsilon_m^{t_1}$ and $\varepsilon_s^{t_1}$ are correlated with a correlation coefficient of ρ.

Clearly, the expectation should be taken over both the stopping time τ and the standard normal variables which govern the random returns. We do not have a closed form expression for the expectation, let alone the optimal market weight. We must resort to numerical procedures. The route we take is a combination of Monte Carlo simulations and numerical maximization. To obtain the expected utility for a given market portfolio weight x_m, we simulate the paths of the total wealth and evaluate the utility function (2.1) on each realized terminal wealth. Each expected utility is averaged over 100,000 realizations. The antithetic variate technique is used to reduce simulation errors.

In generating the terminal wealth, the key step is to simulate the stopping time τ. Given the assumption of independence between the bankruptcy and stock return processes, we can first simulate the stopping time on each path, and then simulate W_1 or W_2 in (2.1) depending on whether the simulated stopping time τ is within or beyond the vesting period. To obtain the stopping time, we draw from a standardized uniform distribution, and compare the random number with p. This is done at the end of each year until for the first time, the uniform random number is smaller than or equal to p. Such a time is the stopping time for this path.

The optimization in (2.1) is carried out using the Brent method in Press, Teukolsky, Vetterling and Flannery (1997). The procedure boils down to numerically searching for the market weight x_m

that maximizes the expected utility. In that search, for each market weight candidate x_m, we need to simulate the expected utility via the procedures described above.

2.2. Definition of Welfare Loss

To study the impact of bankruptcy and holding restrictions, we need to develop some meaningful measures. When studying the impact of uncertainty, the common practice in the literature is to use some versions of certainty equivalent within the utility context. A certainty equivalent is the cash amount replacing the uncertain cash flow but maintaining the same utility level. In our setting, this concept needs to be generalized. Since the employee can only optimize investments in the market portfolio and the risk-free bond, the imposed stock holding is sub-optimal. A natural and logical generalization of the certainty equivalent concept would be to replace the imposed stock holding by a cash amount and then allow the employee to optimally allocate the entire capital between the market portfolio and the risk-free bond. The cash amount thus obtained is a generalized version of "certainty equivalent". The equivalent amount is obviously not certain since it is not placed entirely in the risk-free bond. For this reason, we call this amount the "allocation equivalent", denoted by AE.[6] Since the imposed holding is sub-optimal, by construction, $AE \leq S$. The difference $S - AE$ reflects a welfare loss in the sense of Brennan and Torous (1999) due to either holding restrictions and / or bankruptcy risk.

With the above understanding, we can now precisely define two versions of welfare loss. We first define the "total welfare loss" which is due to both holding restrictions and bankruptcy risk. Here, we replace the stock holding by a cash amount AE_T and then optimally allocate the entire capital amount $W_0 - S + AE_T$ between the market portfolio and the risk-free bond under the condition that the expected utility is equal to that in (2.5). Since we know the optimal allocation rule in this case (i.e., $x_m = \overline{x}_m = \frac{\mu_m - r_f}{\gamma\sigma_m^2}$), AE_T can be solved analytically as

$$AE_T = S - \left[W_0 - e^{\frac{\ln\left[(1-\gamma)U^*\right] \;-\; (1-\gamma)\left[r_f \,+\, \overline{x}_m^2(\mu_m - r_f) \,-\, \frac{1}{2}\gamma\, \overline{x}_m^2\sigma_m^2\right](T_1+T_2)}{1-\gamma}} \right].$$

[6] Cai and Vijh (2004) use a similar definition of certainty equivalent in studying the valuation of executive stocks and options.

The total welfare loss in percentage with respect to the total wealth can then be defined as $loss\_T = \frac{S-AE_T}{W_0} \times 100\%$. The loss can also be defined as a discount to the stock price, i.e., $disc\_T = \frac{S-AE_T}{S} \times 100\%$.

Next, we define the discount due to bankruptcy risk only, $loss\_D$. On the surface, a logical way seems to proceed as follows: calculate the total welfare loss at two bankruptcy probabilities, $loss\_T$ $(p = 0)$ and $loss\_T$ $(p = p')$, and take the difference between the two losses as the loss due to bankruptcy probability p'. However, some careful thinking uncovers a drawback of this approach: the holding restriction is not properly controlled. For $loss\_T$ $(p = 0)$, the holding period is equal to the vesting period T_1; but for $loss\_T$ $(p = p')$, the effective holding period of the stock is shorter than T_1 due to bankruptcy possibilities. Controlling for the effective restricted holding period, the marginal impact of bankruptcy is simply the loss of the stock. Keeping this in mind, we calculate the allocation equivalent (denoted as AE_D) as follows. With the target utility level U^*, we optimize over the market weight x_m similar to the setup in (2.5), except that whenever bankruptcy occurs within the vesting period, we simply liquidate the stock and re–allocate all the wealth according to Merton's myopic rule. This way, the allocation equivalent AE_D is due to the loss of stock value only. The corresponding percentage welfare loss (with respect to the total wealth) is then $loss\_D = \frac{S-AE_D}{W_0} \times 100\%$. Similarly, the discount due to bankruptcy risk can be defined as $disc\_D = \frac{S-AE_D}{S} \times 100\%$. By inference, the loss due to holding restrictions, conditional on the given bankruptcy probability, is $loss\_T - loss\_D$.

3. Impacts of Holding Restrictions and Bankruptcy Risk

3.1. Parameter Values

To begin with, we set the risk-free rate at $r_f = 0.05$ p.a.. As for the market premium $\mu_m - r_f$ and the market volatility σ_m, Browne, Milevsky and Salisbury (2003) set them at 0.06 and 0.2 p.a. respectively according to Ibbotson Associates (2001) while Brennan and Torous (1999) estimate them to be 0.0822 and 0.192 p.a. using the CRSP value-weighted index. We set $\mu_m - r_f = 0.07$

and $\sigma_m = 0.2$ as the base case. The stock's volatility is set at $\sigma_s = 0.4$ p.a. and the correlation at $\rho = 0.45$ to have a base-case beta of 0.9. As for the risk aversion parameter, Brennan and Torous (1999) examine a range of 2 to 7, while Bliss and Panigirtzoglou (2004) estimate γ to be in the range of 3.37 and 9.52 based on S&P500 option prices. We set $\gamma = 4$ as the base case.

The vesting period and the free-allocation period are both set at 10 years: $T_1 = T_2 = 10$. Without loss of generality, the total initial wealth is set $W_0 = \$100$. Brennan and Torous (1999) report that company's own stock accounts for about 30% of a typical defined contribution plan. Holden and VanDerhei (2001) estimate the company stock holding to be 38% among 401(k) plans. These number seem to be consistent with the numbers in Panel B of Table 1. We therefore set $S = \$30$. Finally, to gauge the bankruptcy probability, we use the debt default probability as a proxy. This can be obtained from major rating agencies. According to Altman, Resti and Sironi (2002), for all the rated companies over the period of 1978 to 2001, the annual debt-default rate is 2.95% based on arithmetic averaging and 3.50% based on value-weighted (par value of default) averaging. Therefore a reasonable estimate is between 2% and 4% for an average company.[7] To gain a broader perspective, we examine a range of bankruptcy probabilities for all analyses: $p = [0.00,$ 0.02, 0.04, 0.06, 0.08, 0.10, 0.20, 0.30]. In Section 4, we will use firm-specific bankruptcy probabilities to quantify welfare losses for the companies listed in Table 1.

For comparative analysis, we examine a range for certain parameters. Specifically, $T_1 = [1, 5, 10, 20]$, $T_2 = [0,\ 5,\ 10]$, $\gamma = [2,\ 4,\ 6,\ 20]$, $S = [10,\ 30,\ 50,\ 70]$, $\rho = [0.0,\ 0.3,\ 0.6,\ 0.9]$, $\mu_m - r_f = [0.02, 0.06, 0.10, 0.14]$, and $\sigma_s = [0.1, 0.3, 0.5, 0.7]$.

3.2. Asset Allocation and Welfare Loss: Numerical Results and Implications

3.2.1. Vesting Period, Free Allocation Period, and Bankruptcy Process

To obtain a general sense of the impacts of holding restrictions and bankruptcy risk, we calculate the optimal market weight and welfare losses for different combinations of the vesting period T_1

[7] Of course, it is up to debate whether the debt default probability is an accurate proxy for bankruptcy probability. On the one hand, the bankruptcy rate may be slightly lower than the default rate in that debt default doesn't always lead to bankrutpcy; on the other hand, the true bankruptcy probability may be slightly higher than the default rate since certain factors are not reflected in credit ratings. Again, Enron serves as a good example here.

and the free allocation period T_2. Table 2 reports the results.

The first striking observation is that the optimal asset allocation and welfare losses are practically invariant to the length of the free allocation period T_2. In our setup, once the stock is vested at time t_1, the optimal allocation reverts to Merton's myopic rule, and there is no welfare loss beyond t_1. Therefore, the length of the free allocation period has very little bearing. We will set $T_2 = 10$ years for subsequent analyses.

As for asset allocation within the vesting period, we see that the market weight goes up as the bankruptcy probability p and the vesting period T_1 increase. Moreover, as $p \to 1$ and / or $T_1 \to \infty$, the optimal market weight x_m converges to Merton's myopic proportion, 0.4375. Intuitively, for a given vesting period T_1, as the bankruptcy probability increases, the chance of losing the stock increases; similarly, for a given bankruptcy probability p, as the vesting period increases, the chance of losing the stock also increases. In either case, the stock holding is practically irrelevant in that it is almost certain to be lost before being vested. Thus the employee would allocation her liquid wealth as if the stock was absent. To see this point, when $p = 0.3$ and $T_1 = 20$ years, the probability for the company to go bankrupt is 0.9992 ($= 1 - 0.7^{20}$). In contrast, the probability of the firm going bankrupt within 20 years is only 0.3324 if the annual bankruptcy probability is 0.02.

If the stock and the market portfolio are perfect substitutes (i.e., $\rho = 1$ and $\sigma_s = \sigma_m$), then the optimal market weight is $x_m = 0.1964$. Not surprisingly, $0.1964(0.7) + 0.3 = 0.4375$ (note that we hold 30% of the total wealth in the stock). When the stock and the market portfolio are not perfect substitutes and when the beta (i.e., $\frac{\rho\sigma_s}{\sigma_m}$) is not too high, we always have $x_m > 0.1964$. In other words, we hold the market slightly more, because the stock is a less perfect substitute of the market. Of course, as the bankruptcy probability increases, the employee will increase the market holding to make up the potential loss of the stock.

Turning to welfare losses, we see that the total welfare loss increases as the bankruptcy probability and / or the vesting period increase. As $p \to 1$ and / or $T_1 \to \infty$, the total welfare loss approaches to 30%, the fixed stock holding. These results make perfect intuitive sense. For a vesting period of 10 years, the minimum loss is about 13%. For a typical company with an annual bankruptcy probability between 0.02 and 0.04, the total welfare loss is between 17% and 21% of the total wealth. This translates to a percentage loss of 57% to 70% of the stock value.

The welfare loss due to bankruptcy risk shares the same patterns as the total welfare loss, which again makes intuitive sense: the chance of losing the stock increases as the bankruptcy probability and the vesting period increase. For a typical company with a vesting period of 10 years and an annual bankruptcy probability between 0.02 and 0.04, the welfare loss is between 6% and 10% of the total wealth, which translates to between 20% and 33% of the stock value. This magnitude is by no means trivial.

Incidentally, it can be seen that the welfare loss due to holding restrictions is positively related to the vesting period, but negatively related to the bankruptcy probability. Both results make intuitive sense. Naturally, when the bankruptcy probability is high, the total welfare loss is almost exclusively due to bankruptcy risk.

So far, all the results are based on an annual frequency of the company's status revelation. Although seemingly restrictive, this assumption is not too unrealistic at all. More often than not, major revelations of a firm's operation come out at annual meetings or in annual financial statements. Nonetheless, it is important to know if a more frequent revelation will make a difference. To this end, we repeat the calculations in Table 2 with one modification: we allow quarterly revelations and convert the annual bankruptcy probability into quarterly. For an annual bankruptcy probability p, the corresponding quarterly probability is $1 - (1-p)^{1/4}$. For brevity, we omit the table and briefly outline numerical results below.[8]

To start with, the free allocation period T_2 again has very little bearing on either the optimal market weight or the welfare loss. More importantly, comparing with the results in Table 2, there are no discernible differences in the optimal market weight and total welfare loss when the vesting period is beyond one year. However, the welfare loss due to bankruptcy risk is slightly higher with quarterly revelations. The overall results make intuitive sense. When the revelation frequency increases, the chance of detecting an early bankruptcy and hence losing the stock increases. Therefore the welfare

[8]Once again, trading is always continuous in our setting, regardless of the frequency of bankrutpcy revelations.

loss due to bankruptcy risk increases. But this increased bankruptcy risk will also reduce the effective holding period, alleviating the welfare loss due to holding restrictions. This is why we fail to observe a discernible change in the total welfare loss. Nevertheless, the increase in welfare loss due to bankruptcy risk is not very significant in magnitude. For example, with a vesting period of 20 years, a free allocation period of 10 years, and an annual bankruptcy probability of 0.02, the loss is 7.83% with quarterly revelation, compared with 7.71% with annual revelation. For the remaining analyses, we use the annual revelation setting.

We can summarize the results to this point and draw the following conclusions. 1) within the vesting period, the weight on the market portfolio is higher, the longer the vesting period and / or the higher the bankruptcy probability. Participants of 401(k) plans should make their allocations accordingly. 2) Both the total welfare loss and that due to bankruptcy risk are positively related to the length of the vesting period and the bankruptcy probability, and the magnitude is significant with calibrated parameter values. For a typical firm, the total welfare loss is 57% of the stock value when the vesting period is 10 years and the annual bankruptcy probability is 0.02. In what follows, we show the results of comparative static analysis for other parameters.

3.2.2. Risk Aversion, Equity Premium, and Abnormal Stock Returns

Undoubtedly, an employee's risk aversion and the equity market conditions will affect her asset allocation decisions and the welfare loss. For the risk aversion parameter, we examine a range of [2, 4, 6, 20], with $\gamma = 20$ being the case of high risk aversion. Panel A of Table 3 contains the results. To start with, we see that for each level of risk aversion, the patterns of optimal market weights and welfare losses are the same as those in Table 2. In particular, the market weight approaches Merton's myopic constant as the bankruptcy probability and / or the vesting period increase, and the total welfare loss approaches the stock weighting of 30% in those cases.

As the risk aversion gets higher, the total welfare loss increases, which makes intuitive sense. However, it does not seem to take a very high risk aversion to incur a big welfare loss. For instance, when the bankruptcy probability is higher than 0.06, the total welfare losses under $\gamma = 6$ and $\gamma = 20$

are both very high and not that different. Interestingly though, the welfare loss due to bankruptcy risk is smaller when the risk aversion is higher, and this relation is stronger when the bankruptcy probability is high. To understand this, notice first that, when the bankruptcy risk is absent (i.e., $p = 0$), welfare loss increases dramatically as risk aversion increases. The higher the risk aversion, the more costly the sub-optimal allocation. With this in mind, for a given non-zero bankruptcy probability, since the loss portion due to misallocation increases as risk aversion increases yet the chance of losing the stock remains the same, the welfare loss due to bankruptcy risk will naturally go down. In other words, as risk aversion increases, holding restrictions become more and more detrimental relative to bankruptcy risk. Regardless, a higher risk aversion is associated with a higher welfare loss, and this relation is stronger when the bankruptcy risk is low.

Another important factor for asset allocation is the size of equity premium. To see the impact of this factor, we examine a range of [0.02, 0.06, 0.10, 0.14] for $\mu_m - r_f$. Panel B of Table 3 reports the results. We see that a higher equity premium induces more holding of the market, consistent with the Merton's myopic rule. The optimal holding of the market seems to respond more to the bankruptcy probability when the equity premium is low. As the equity premium goes up, the total welfare loss goes down while the welfare loss due to bankruptcy risk takes a humped shape, albeit the magnitude of changes is small in both cases. The intuition for the total welfare loss to be lower when the equity premium is high is that, the more attractive equity market can more than offset the loss.

Just as the market portfolio's characteristics affect the employee's asset allocation decisions and welfare losses, the stock's profile will also do. So far, we have assumed a CAPM relationship. What if the company's stock under- or out-perform the market? To address this question, we repeat the calculations for a range of abnormal returns on the stock. Denoted by α, this abnormal return is the difference between the actual expected return and that dictated by the CAPM. The results are reported in Panel C of Table 3.

As the abnormal return gets bigger, the total welfare loss gets smaller while that due to bankruptcy risk gets bigger. A bigger abnormal return leads to a smaller total welfare loss because the stock's attractiveness can more than offset the holding restrictions; at the same time, it leads to a bigger welfare loss due to bankruptcy risk because other things being equal, the loss is more costly when the asset is more attractive. As for the optimal market weight, when the bankruptcy risk is low, higher abnormal returns are related to lower market weights. This makes sense because the attractive stock holding gets a decent exposure to risky returns; hence the reliance on the market portfolio becomes less.

The above observations have some interesting implications regarding the consequence of employees' subjective assessment of their own stocks. What if the employee is more optimistic than the market about her own stock? In fact, anecdotal evidence suggests that many employees voluntarily hold company stocks in their 401(k) plans simply because they consider their company stocks being more valuable. Panel C of Table 3 indicates that employees would invest less in the market portfolio when the stock return is perceived to be higher. Therefore, a falsely optimistic assessment of the company stock will reduce the proper exposure to the equity market outside of the restricted holding.

3.2.3. Company Stock's Volatility and Correlation with the Market

Other aspects of the stock's profile are its volatility and correlation with the market. They determine the total risk as well as the systematic risk of the stock. We repeat the calculations by varying the volatility between 0.1 and 0.7. Panel A of Table 4 contains the results. It is seen that the optimal holding of the market portfolio generally goes down as the stock's volatility increases. In fact, when the stock's volatility is very low (e.g., $\sigma_s = 0.1$), the optimal weight is higher than Merton's myopic weight (0.4375). Intuitively, as the stock's risk increases, the exposure to equity risk increases, hence the market holding is reduced to maintain a balanced total exposure to equity risk.

As for welfare losses, the total welfare loss increases as the stock's volatility increases, and this relation is much stronger when the bankruptcy risk is low. To understand the relationship, we need to realize that, unlike the market portfolio, the stock's total risk is not completely compensated by its expected return. For every unit of increase in the volatility, only a portion of it (i.e., $\frac{\rho\sigma_s}{\sigma_m}$)

commends a compensation. Therefore, a higher volatility exacerbates the negative impact of holding restrictions, leading to a bigger total welfare loss. In contrast, the welfare loss due to bankruptcy risk goes down as the stock's volatility increases. The reason is related to the above explanations. Since the increased volatility is only partially compensated, the loss in the stock will not increase dramatically because the expected return did not increase dramatically; relatively speaking, the cost of holding restrictions is much higher.

We now examine the impact of correlation which is varied between 0.0 and 0.9. Panel B of Table 4 contains the results. For the optimal market holding, correlation and volatility have very similar effects, albeit for very different reasons. When the correlation is zero, the optimal weight is higher than the myopic weight. Here, the stock's expected return is the risk-free rate, and the extra holding of the market is to make up the sub-optimal holding. As the correlation increases, the market holding is reduced, and the reduction is dramatic when the bankruptcy risk is low. This makes perfect intuitive sense, since the stock becomes a closer substitute of the market as correlation increases. In the absence of bankruptcy risk, the market holding goes from 41.6% to 31.5% of the liquid wealth when the correlation moves from 0.3 to 0.6.

For all levels of bankruptcy risk, the total welfare loss goes down as correlation increases. Again, this is due to the substitution effect. When the stock is a closer substitute of the market, holding restrictions do not cause too much welfare loss (when the imposed holding is below the myopic weight). The welfare loss due to bankruptcy risk increases slightly with the correlation. The reason is similar to the above. When the stock is a close substitute of the market, losing the stock is more devastating because it requires a more dramatic change in allocation. Regardless, the response of welfare loss to the level of correlation is not very strong.

To summarize, the company stock's correlation with the market has more bearing on optimal allocation than on welfare losses; the stock's volatility has much bearing on both. When the volatility is high, total welfare loss is high; this relationship is stronger when the bankruptcy risk is low. Our results are consistent with the findings of Brown, Liang and Weisbenner (2004) who report that less risky firms (characterized by lower volatility and lower expected bankruptcy rate) are more likely to provide the employer match in company stock in pension contributions. Our results clearly imply that, risky firms should shy away from imposing company stocks when contributing to employees' 401(k) plans.

3.2.4. Size of Imposed Stock Holding

All of our results so far are based on an average level of stock holding of 30%. As apparent in Table 1, the stock holding does vary across firms. To see the complete picture, we now repeat the calculations for four levels of stock holdings: $S/W_0 = 0.1$, 0.3, 0.5 and 0.7. The results are reported in Table 5. Here, we also report the total welfare loss as a percentage of the stock price. This measure can be interpreted as the percentage discount of the stock value due to both holding restrictions and bankruptcy risk (hence denoted by "disc" in the table).

Aside from the familiar patterns already observed in Table 2, we see that the optimal market weight is generally smaller and the welfare losses become larger when S/W_0 is high, as one would expect. More importantly, as the imposed holding increases, the percentage discount also increases. The magnitude of the discount is large. For instance, when the imposed stock holding is equal to or more than 50% of the total wealth, the minimum discount is around 55%, regardless of the bankruptcy risk level. If the bankruptcy probability is 0.02 per year and the imposed holding is more than 50%, then at least 70% of the stock value is discounted away. This is a significant welfare loss.

4. Empirical Estimation of Welfare Losses

In this section, we attempt to quantify the impacts of holding restrictions and bankruptcy risk on the pension plans listed in Table 1. December 29 of 2000 is chosen as the ending date for welfare loss calculations, partly because Enron went bankrupt in December 2001. We would like to shed some empirical light on welfare issues without the benefit of hindsight.

We collect daily returns for all the stocks in Table 1 and the S&P500 index over the period of January 2, 1996 to December 29, 2000. The annualized volatility and the correlation between the stock return and the market index are then estimated for each stock, as reported in Table 6. The annualized annual return on the S&P500 index is 16.29%, which is used to proxy the expected return on the market portfolio. The annualized standard deviation for the S&P500 index is estimated to be 0.1837. We average the daily 91-day T-bill rates in 2000 and use this average (6.01%) to proxy the risk-free rate. The expected return on each stock is then calculated via CAPM with the above parameter estimates.

Another critical element to estimate is the bankruptcy probability. Similar to our simulations, we use the debt default probability as a proxy. To this end, we collect from *Datastream* the credit rating, by both Moody's and Standard & Poor's, for each company as of 2000. (In this process, we have to drop ADC Telecom since it was not rated.) For each rating, Moody's provides the average annual default rate and the maximum annual default rate for the period of 1980-2000 (Moody's, 2001). To reflect the impact of severe credit conditions without too much exaggeration, we choose the middle point between the annual average and the annual maximum default rates as the bankruptcy proxy. Whenever the two rating agencies render different ratings, we choose the lower (i.e., the riskier) denomination. The final estimates are presented in Table 6.

The percentage of pension investments in company stock, i.e., S/W_0, is taken from Table 1. The values for the remaining parameters are the same as before, i.e., $\gamma = 4.0$, $T_1 = T_2 = 10$ years. With all the parameter values in place, we then proceed to calculate the optimal allocation and welfare losses. The last three columns of Table 6 presents the results. For ease of discussion, we sort the companies first by the default rate (or bankruptcy probability) then by the stock holding.

There are several interesting observations. First, except for Chiquita Brands, the optimal holding of the market portfolio for all companies is lower than the Merton's myopic constant, 0.7622. Given a bankruptcy probability, a higher stock holding causes a lower optimal holding of the market, consistent with our previous findings. Second, the total discount ranges from 58.38% to 99.2%, averaging at 79.02%, while the discount due to bankruptcy risk ranges from zero to 76.14%, averaging at 12.69%. Third, consistent with Table 4, given the same bankruptcy probability and similar stock holdings, a lower volatility usually leads to a smaller total discount but a bigger discount due to bankruptcy risk. Polaroid and Providian Financial are good examples.

Admittedly, the market conditions and stock's profiles are different at different times. For instance, at the end of 2000, Enron was still rated at the BBB level. Suppose the rating was at the BB level (with a bankruptcy probability of 3.46%) while all other parameters retained the same values, then the total discount and the discount due to bankruptcy risk would have been 81.39% and 21.26% respectively; the corresponding numbers would have been 94.34% and 53.60% had the rating been at the B level. In summary, our simple empirical investigation does reveal the severe welfare losses due to holding restrictions and bankruptcy risk.

5. Summary and Conclusions

This paper studies the impacts of holding restrictions and bankruptcy risk on 401(k) investments in firm's own stock. Using a framework of maximizing the end-of-period expected utility, we study both the optimal asset allocation for the liquid wealth and welfare losses due to imposed stock holdings and the chance for the company to go bankrupt. The asset allocation is done in a dynamic manner (as opposed to the static buy-and-hold), and welfare losses are calculated by generalizing the concept of certainty equivalent. The stock and market values are modelled as a joint geometric Brownian motion, and the bankruptcy probability is assumed to be time-homogenous and modelled as a uniform distribution. The optimization (in solving for the optimal market weight and certainty equivalent) is carried out using a combination of Monte Carlo simulations and numerical maximization.

We show that the holding restrictions and bankruptcy risk can affect optimal allocations and cause significant welfare losses. The specific findings can be summarized as follows. 1) When the bankruptcy risk is very high and / or when the vesting period is very long, the optimal weight on the market portfolio during the vesting period is close to Merton's myopic weight. Otherwise, it is generally lower than the myopic weight. 2) Welfare losses are higher when the bankruptcy risk is high and / or the vesting period is long. The magnitude is significant. With calibrated parameters, our simulations show that the welfare loss can be 50% of the stock value or higher. 3) Welfare losses are bigger when the risk aversion is high and / or the stock's abnormal return is low. Surprisingly, the market equity premium does not have a significant bearing on welfare losses. 4) The stock's volatility has much bearing on both the optimal allocation and welfare losses. When the volatility and bankruptcy risk are high, the welfare loss is large. 5) As expected, the welfare loss is larger, the bigger the imposed stock holdings. 6) Estimates of welfare losses for a sample of companies corroborate the major findings.

The upshot of our analysis is that, investing company's own stock in employees' 401(k) plans is very costly. The welfare loss due to sub-optimal allocations is substantial. This welfare loss is exacerbated by the bankruptcy risk. The contribution of bankruptcy risk to the total welfare loss is significant based on calibrated parameter values. Specifically, for a 10-year vesting period, the bankruptcy risk alone can reduce the stock value by 20 to 30%.

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Table 1: List of Companies Which Contribute Company Stocks to 401(k) Plans

Panel A: Large Hybrid 401(k) and ESOP Plans

Company	% of DC Plan in Company Stock	Estimated ESOP Deduction from EGTRRA ($ millions)
Abbott Laboratories	82%	$28
Anheuser-Busch	83%	$15
Bank of America	43%	$8
Ford Motor	50%	$90
Marsh & McLennan	61%	$10
McDonalds	74%	$4
Pfizer	82%	$23
Procter & Gamble	92%	$127
SBC	38%	$56
Verizon	51%	$31

Source: Schulz, E. E. and T. Francis, 2002, Companies' Hot Tax Break: 401(k)s, Wall Street Journal, January 31, 2002, p. C1.

Panel B: Performance of Company Stock in 401(k) Pension Plans

Company	% of DC Plan in Company Stock	% Stock Price Change during Mar. 2000 - Dec. 2001
Polaroid	19%	-99.6%
Enron	41%	-99.6%
Global Crossing	16%	-97.5%
Weirton	16%	-96.4%
Crown Cork & Seal	11%	-92.5%
Providian Financial	19%	-91.8%
KS City Southern	80%	-91.8%
Lucent Technologies	16%	-89.2%
Owens Corning	25%	-88.5%
Montana Power	25%	-88.0%
Northern Telecom	30%	-86.6%
Corning	32%	-86.0%
W.R. Grace	11%	-84.3%
Chiquita Brands	11%	-82.8%
ADC Telecom	46%	-80.4%

Source: Farrell, C., 2002, The Problem with Pension Plans, Business Week Online, Sound Money 1/11/02, New York: McGraw Hill, www.businessweek.com/bwdaily/dnflash/jan2002 /nf20020111_3044.htm.

Note:
"DC" stands for Defined Contribution, "ESOP" for Employee Stock Ownership Plan, and "EGTRRA" for Economic Growth and Taxpayer Relief Reconciliation Act.

Table 2: Optimal Allocation and Welfare Loss versus Vesting Period

	T_1 = 1 year			T_1 = 5 years			T_1 = 10 years			T_1 = 20 years		
prob.	x_m	loss_T	loss_D	x_m	loss_T	loss_D	x_m	loss_T	loss_D	x_m	loss_T	loss_D
					Panel A, T_2 = 0 years							
0.00	0.274	2.12	0.00	0.337	8.25	0.00	0.369	13.04	0.00	0.397	18.59	0.00
0.02	0.290	3.21	1.11	0.355	11.68	4.01	0.385	17.47	5.91	0.411	23.11	7.76
0.04	0.302	4.27	2.19	0.368	14.44	7.26	0.397	20.51	10.19	0.418	25.77	12.57
0.06	0.314	5.28	3.23	0.379	16.69	9.95	0.406	22.71	13.33	0.424	27.27	15.84
0.08	0.324	6.29	4.26	0.387	18.58	12.18	0.413	24.38	15.79	0.428	28.31	18.16
0.10	0.332	7.22	5.21	0.395	20.15	14.10	0.417	25.63	17.73	0.434	28.78	19.86
0.20	0.365	11.37	9.45	0.413	25.21	20.41	0.431	28.72	23.17	0.433	29.90	24.01
0.30	0.385	14.90	13.06	0.426	27.71	23.81	0.433	29.67	25.42	0.438	29.91	25.62
					Panel B, T_2 = 5 years							
0.00	0.275	2.11	0.00	0.337	8.24	0.00	0.370	13.02	0.00	0.397	18.56	0.00
0.02	0.290	3.21	1.12	0.356	11.66	4.00	0.385	17.44	5.91	0.411	23.07	7.73
0.04	0.302	4.27	2.20	0.369	14.42	7.25	0.397	20.52	10.18	0.417	25.74	12.55
0.06	0.313	5.27	3.22	0.380	16.67	9.93	0.406	22.68	13.31	0.425	27.21	15.83
0.08	0.323	6.28	4.26	0.387	18.55	12.17	0.414	24.36	15.78	0.428	28.25	18.16
0.10	0.330	7.20	5.20	0.396	20.13	14.09	0.419	25.58	17.72	0.434	28.76	19.81
0.20	0.362	11.37	9.45	0.414	25.17	20.41	0.432	28.69	23.14	0.435	29.85	23.99
0.30	0.384	14.89	13.05	0.426	27.69	23.80	0.434	29.64	25.40	0.438	29.88	25.60
					Panel C, T_2 = 10 years							
0.00	0.275	2.09	0.00	0.338	8.22	0.00	0.370	12.98	0.00	0.397	18.51	0.00
0.02	0.291	3.19	1.12	0.357	11.62	3.99	0.386	17.40	5.91	0.412	23.00	7.71
0.04	0.302	4.25	2.21	0.370	14.38	7.24	0.398	20.49	10.17	0.418	25.68	12.53
0.06	0.313	5.24	3.22	0.381	16.63	9.91	0.407	22.64	13.29	0.425	27.14	15.80
0.08	0.323	6.26	4.26	0.388	18.51	12.17	0.414	24.34	15.78	0.429	28.20	18.15
0.10	0.329	7.18	5.20	0.397	20.10	14.09	0.420	25.53	17.70	0.435	28.72	19.77
0.20	0.361	11.35	9.46	0.415	25.13	20.40	0.434	28.64	23.12	0.436	29.81	23.97
0.30	0.384	14.87	13.05	0.427	27.66	23.79	0.435	29.60	25.38	0.438	29.84	25.58

Note:

1. For each combination of bankrutcy probability (prob.) and vesting period (T_1), this table reports the optimal holding of the market (x_m) relative to the liquid wealth, the total welfare loss (loss_T) due to both holding restrictions and bankrutcy risk, and the welfare loss (loss_D) due to bankrutcy risk only. Each panel corresponds to a different T_2 which is the period between the end of the vesting and the end of the planning horizon or time of retirement. Both "loss_T" and "loss_D" are expressed as percentage of the total wealth. Please see Section 2 for the precise procedures of calculating the two percentage losses.
2. The bankruptcy probability is per year, and we assume that the firm's status is revealed at the end of each year.
3. The optimal proportion in the market is 0.4375 when holding restrictions and bankruptcy risk are absent, and it is invariant to the planning horizon.
4. Values of other parameters: $W_0 = \$100,\ S = \$30,\ r_f = 0.05,\ \mu_m = 0.12,\ \sigma_m = 0.2,$ $\sigma_s = 0.4,\ \rho = 0.45,\ \gamma = 4.0,$ and $\mu_s = r_f + \sigma_s \rho / \sigma_m [\mu_m - r_f]$.

Table 3: Optimal Allocation and Welfare Loss versus Risk Aversion and Equity Returns

Panel A: Risk Aversion

prob.	γ = 2.0			γ = 4.0			γ = 6.0			γ = 20.0		
	x_m	loss_T	loss_D	x_m	loss_T	loss_D	x_m	loss_T	loss_D	x_m	loss_T	loss_D
0.00	0.872	8.76	0.00	0.370	12.98	0.00	0.219	15.52	0.00	0.040	21.78	0.00
0.02	0.872	13.54	5.90	0.386	17.40	5.91	0.240	19.72	5.81	0.066	25.29	5.01
0.04	0.872	17.19	10.47	0.398	20.49	10.17	0.253	22.41	9.73	0.074	26.76	7.36
0.06	0.873	19.91	13.93	0.407	22.64	13.29	0.263	24.21	12.55	0.079	27.62	8.96
0.08	0.874	22.10	16.72	0.414	24.34	15.78	0.270	25.59	14.78	0.081	28.24	10.22
0.10	0.875	23.72	18.91	0.420	25.53	17.70	0.276	26.54	16.50	0.084	28.63	11.22
0.20	0.876	28.10	24.93	0.434	28.64	23.12	0.289	28.96	21.53	0.088	29.60	14.58
0.30	0.877	29.47	27.12	0.435	29.60	25.38	0.290	29.69	23.87	0.088	29.88	16.79
Merton	0.875			0.438			0.292			0.088		

Panel B: Market Equity Premium

prob.	$\mu_m - r_f = 0.02$			$\mu_m - r_f = 0.06$			$\mu_m - r_f = 0.10$			$\mu_m - r_f = 0.14$		
	x_m	loss_T	loss_D	x_m	loss_T	loss_D	x_m	loss_T	loss_D	x_m	loss_T	loss_D
0.00	0.016	13.54	0.00	0.299	13.09	0.00	0.584	12.68	0.00	0.856	12.47	0.00
0.02	0.042	17.85	5.86	0.317	17.49	5.90	0.593	17.16	5.95	0.855	16.89	6.01
0.04	0.060	20.83	10.05	0.330	20.56	10.16	0.600	20.28	10.16	0.860	19.96	9.83
0.06	0.074	22.97	13.19	0.340	22.72	13.28	0.611	22.34	13.29	0.889	21.41	13.15
0.08	0.083	24.58	15.60	0.347	24.39	15.76	0.614	24.09	15.78	0.872	23.48	15.30
0.10	0.091	25.80	17.53	0.353	25.60	17.68	0.621	25.20	17.68	0.893	24.31	17.38
0.20	0.113	28.82	22.97	0.369	28.70	23.11	0.633	28.34	23.14	0.907	27.32	23.11
0.30	0.120	29.69	25.23	0.371	29.64	25.37	0.627	29.39	25.34	0.891	28.57	25.06
Merton	0.125			0.375			0.625			0.875		

Panel C: Abnormal Return on the Stock

prob.	α = - 0.04			α = - 0.02			α = 0.02			α = 0.04		
	x_m	loss_T	loss_D	x_m	loss_T	loss_D	x_m	loss_T	loss_D	x_m	loss_T	loss_D
0.00	0.385	17.40	0.00	0.378	15.34	0.00	0.362	10.29	0.00	0.354	7.23	0.00
0.02	0.395	20.41	4.72	0.391	18.98	5.27	0.382	15.66	6.66	0.378	13.76	7.52
0.04	0.403	22.63	8.35	0.400	21.60	9.20	0.395	19.29	11.26	0.393	18.02	12.49
0.06	0.411	24.21	11.13	0.409	23.46	12.15	0.406	21.78	14.55	0.405	20.87	15.95
0.08	0.417	25.50	13.42	0.415	24.93	14.55	0.413	23.71	17.13	0.413	23.05	18.59
0.10	0.421	26.41	15.26	0.420	25.98	16.43	0.419	25.06	19.07	0.419	24.57	20.54
0.20	0.434	28.88	20.83	0.434	28.76	21.95	0.434	28.52	24.35	0.434	28.40	25.63
0.30	0.435	29.66	23.44	0.435	29.63	24.39	0.435	29.57	26.40	0.435	29.54	27.45

Note:

1. For each parameter combination, this table reports the optimal holding of the market (x_m) relative to the liquid wealth, the total welfare loss (loss_T) due to both holding restrictions and bankrutcy risk, and the welfare loss (loss_D) due to bankrutcy risk only. Both "loss_T" and "loss_D" are expressed as percentage of the total wealth. Please see Section 2 for the precise procedures of calculating the two percentage losses.
2. The abnormal return on the stock is defined as $\alpha = \mu_s^{actual} - \mu_s$, where μ_s^{actual} is the actual expected return on the stock and $\mu_s = r_f + \sigma_s\rho/\sigma_m[\mu_m - r_f]$.
3. The bankruptcy probability is per year, and we assume that the firm's status is revealed at the end of each year.
4. The numbers in the row with the heading "Merton" are optimal market weights when holding restrictions and bankruptcy risk are absent. For Panel C, it is 0.4375.
5. Values of base parameters: $W_0 = \$100$, $S = \$30$, $r_f = 0.05$, $\mu_m = 0.12$, $\sigma_m = 0.2$, $\sigma_s = 0.4$, $\rho = 0.45$, $\gamma = 4.0$, $T_1 = T_2 = 10$ years, and $\mu_s = r_f + \sigma_s\rho/\sigma_m[\mu_m - r_f]$.

Table 4: Optimal Allocation and Welfare Loss versus Stock Profiles

Panel A: Stock Volatility

prob.	$\sigma_s = 0.1$ x_m	loss_T	loss_D	$\sigma_s = 0.3$ x_m	loss_T	loss_D	$\sigma_s = 0.5$ x_m	loss_T	loss_D	$\sigma_s = 0.7$ x_m	loss_T	loss_D
0.00	0.520	1.29	0.00	0.389	8.83	0.00	0.367	16.66	0.00	0.377	22.23	0.00
0.02	0.486	10.32	9.24	0.403	14.71	7.06	0.381	19.91	4.89	0.387	23.93	3.32
0.04	0.473	15.69	14.77	0.412	18.62	11.84	0.392	22.28	8.61	0.395	25.24	6.08
0.06	0.464	19.30	18.51	0.419	21.30	15.24	0.401	23.96	11.42	0.403	26.20	8.27
0.08	0.459	21.92	21.21	0.424	23.36	17.87	0.409	25.31	13.73	0.409	27.00	10.15
0.10	0.455	23.75	23.17	0.428	24.80	19.83	0.415	26.27	15.56	0.414	27.58	11.73
0.20	0.446	28.18	27.85	0.437	28.45	25.08	0.432	28.84	21.03	0.431	29.20	16.94
0.30	0.438	29.49	29.24	0.436	29.56	27.02	0.434	29.65	23.56	0.434	29.74	19.77

Panel B: Stock's Correlation with the Market

prob.	$\rho = 0.0$ x_m	loss_T	loss_D	$\rho = 0.3$ x_m	loss_T	loss_D	$\rho = 0.6$ x_m	loss_T	loss_D	$\rho = 0.9$ x_m	loss_T	loss_D
0.00	0.495	14.66	0.00	0.416	13.76	0.00	0.315	11.87	0.00	0.145	7.88	0.00
0.02	0.482	18.52	5.44	0.421	17.89	5.65	0.347	16.76	6.33	0.249	14.80	8.11
0.04	0.473	21.28	9.43	0.424	20.82	9.79	0.368	20.08	10.76	0.299	18.91	13.07
0.06	0.467	23.21	12.42	0.428	22.87	12.85	0.384	22.37	13.96	0.332	21.64	16.45
0.08	0.462	24.76	14.82	0.431	24.50	15.31	0.396	24.14	16.48	0.356	23.66	19.01
0.10	0.459	25.84	16.74	0.434	25.65	17.22	0.405	25.40	18.40	0.373	25.07	20.90
0.20	0.448	28.73	22.19	0.439	28.67	22.67	0.429	28.61	23.75	0.419	28.54	25.86
0.30	0.439	29.63	24.57	0.436	29.61	24.99	0.433	29.59	25.93	0.430	29.58	27.72

Note:
1. For each combination of bankrutcy probability (prob.) and stock volatility (σ_s) / correlation with the market (ρ), this table reports the optimal holding of the market (x_m) relative to the liquid wealth, the total welfare loss (loss_T) due to both holding restrictions and bankrutcy risk, and the welfare loss (loss_D) due to bankrutcy risk only. Both "loss_T" and "loss_D" are expressed as percentage of the total wealth. Please see Section 2 for the precise procedures of calculating the two percentage losses.
2. The bankruptcy probability is per year, and we assume that the firm's status is revealed at the end of each year.
3. The optimal proportion in the market is 0.4375 when holding restrictions and bankruptcy risk are absent, and it is invariant to the volatility and correlation.
4. Values of base parameters: $W_0 = \$100$, $S = \$30$, $r_f = 0.05$, $\mu_m = 0.12$, $\sigma_m = 0.2$, $\sigma_s = 0.4$, $\rho = 0.45$, $\gamma = 4.0$, $T_1 = T_2 = 10$ years, and $\mu_s = r_f + \sigma_s\rho/\sigma_m[\mu_m - r_f]$.

Table 5: Optimal Allocation and Welfare Loss versus Imposed Stock Holding

	$S/W_0 = 0.1$				$S/W_0 = 0.3$				$S/W_0 = 0.5$				$S/W_0 = 0.7$			
prob.	x_m	loss_T	loss_D	disc	x_m	loss_T	loss_D	disc	x_m	loss_T	loss_D	disc	x_m	loss_T	loss_D	disc
0.00	0.409	2.37	0.00	23.65	0.370	12.98	0.00	43.27	0.332	27.68	0.00	55.35	0.279	46.08	0.00	65.83
0.02	0.413	3.95	1.81	39.54	0.386	17.40	5.91	58.01	0.370	34.96	11.59	69.92	0.370	56.42	21.28	80.60
0.04	0.417	5.20	3.28	52.04	0.398	20.49	10.17	68.31	0.391	39.23	18.68	78.46	0.397	60.91	31.12	87.01
0.06	0.421	6.14	4.41	61.38	0.407	22.64	13.29	75.47	0.404	41.96	23.55	83.93	0.412	63.49	37.27	90.70
0.08	0.425	6.93	5.36	69.31	0.414	24.34	15.78	81.12	0.413	43.97	27.24	87.93	0.420	65.23	41.68	93.19
0.10	0.427	7.50	6.10	74.96	0.420	25.53	17.70	85.10	0.420	45.34	30.02	90.67	0.426	66.38	44.95	94.83
0.20	0.436	9.14	8.24	91.37	0.434	28.64	23.12	95.48	0.435	48.66	37.82	97.32	0.437	69.00	54.05	98.58
0.30	0.436	9.71	9.04	97.11	0.435	29.60	25.38	98.68	0.435	49.63	41.32	99.25	0.437	69.73	58.44	99.61

Note:

1. For each combination of bankrutcy probability (prob.) and the proportion of imposed stock holding (S/W_0), this table reports the optimal holding of the market (x_m) relative to the liquid wealth, the total welfare loss (loss_T) due to both holding restrictions and bankrutcy risk, the welfare loss (loss_D) due to bankrutcy risk only, and the discount (disc) of the stock value due to both holding restrictions and bankruptcy risk. Both "loss_T" and "loss_D" are expressed as percentage of the total wealth; while "disc" is expressed as percentage of the stock price. Please see Section 2 for the precise procedures of calculating the percentage numbers.
2. The bankruptcy probability is per year, and we assume that the firm's status is revealed at the end of each year.
3. The optimal proportion in the market is 0.4375 when holding restrictions and bankruptcy risk are absent.
4. Values of other parameters: $W_0 = \$100,\ r_f = 0.05,\ \mu_m = 0.12,\ \sigma_m = 0.2,$ $\sigma_s = 0.4,\ \rho = 0.45,\ \gamma = 4.0,\ T_1 = T_2 = 10$ years, and $\mu_s = r_f + \sigma_s\rho/\sigma_m[\mu_m - r_f]$.

Table 6: Empirical Estimation of Asset Allocation and Welfare Losses

Company	Credit Rating Moody's	Credit Rating S&P	Annual Default Rate (%)	Stock Holding S / W_0 (%)	Stock Volatility σ_s	Correlation ρ	x_m	disc_T (%)	disc_D (%)
Pfizer	Aaa	AAA	0.000	82.0	0.667	0.249	0.7005	87.15	0.00
SBC	Aa3	AA	0.135	38.0	0.439	0.287	0.7225	63.22	1.11
Abbott Laboratories	Aa1	AAA	0.135	82.0	0.438	0.322	0.6996	69.33	1.43
Procter & Gamble	Aa2	AA	0.135	92.0	0.472	0.282	0.6936	75.28	1.25
Northern Telecom	A2	A	0.320	30.0	0.744	0.373	0.7211	80.14	1.65
Corning	A2	A	0.320	32.0	0.688	0.308	0.7226	80.51	1.65
Bank of America	Aa1	AA-	0.320	43.0	0.488	0.470	0.7046	58.38	3.26
Ford Motor	A2	A	0.320	50.0	0.431	0.353	0.7176	62.44	3.19
Verizon	A1	A+	0.320	51.0	0.445	0.257	0.7203	69.48	2.67
Anheuser-Busch	A1	A+	0.320	83.0	0.507	0.134	0.7108	82.98	2.07
Crown Cork & Seal	Baa2	BBB	0.745	11.0	0.460	0.181	0.7471	62.90	4.70
Lucent Technologies	A3	BBB+	0.745	16.0	0.666	0.341	0.7313	72.09	4.16
Enron	Bba3	BBB	0.745	41.0	0.466	0.215	0.7245	73.01	5.11
Marsh & McLennan	Baa2	BBB	0.745	61.0	0.492	0.381	0.7024	70.94	6.52
McDonalds	Baa2	BBB	0.745	74.0	0.448	0.270	0.7110	74.44	6.48
W.R. Grace	WR	BB+	3.460	11.0	0.966	0.154	0.7588	92.37	9.08
Global Crossing	Ba2	BB+	3.460	16.0	0.978	0.386	0.7479	89.57	11.71
Polaroid	Ba3	BBB	3.460	19.0	0.478	0.331	0.7412	70.94	23.37
Providian Financial	Ba1	BB+	3.460	19.0	0.669	0.304	0.7399	82.50	16.65
KS City Southern	Baa2	BB	3.460	80.0	1.376	0.159	0.7610	99.20	3.83
Weirton	B1	B-	14.745	16.0	0.742	0.099	0.7570	95.05	34.97
Owens Corning	Ba2	BB-	14.745	25.0	0.843	0.173	0.7554	96.59	31.38
Montana Power	B2	BB-	14.745	25.0	0.463	0.164	0.7553	93.04	52.29
Chiquita Brands	Caa1	B-	39.320	11.0	0.467	0.131	0.7762	94.84	76.14

Note:

1. Companies are from the combined lists in Table 1. For each company, we report the credit ratings, the annual default rate for that rating, the stock holding in the 401(k) plan (S/W_0), the annualized volatility (σ_s), the correlation with the market (ρ), the optimal holding of the market (x_m) relative to the liquid wealth, the total discount (disc_T) due to both holding restrictions and bankrutcy risk, and the discount (disc_D) due to bankrutcy risk only. Both "disc_T" and "disc_D" are expressed as percentage of the stock's market value. Please see Section 2 for the precise procedures of calculating the percentage numbers.
2. Section 4 explains the estimation of the annual default rate, the volatility and the correlation. The market portfolio is proxied by the S&P500 index. The sample period is from January 2, 1996 to December 29, 2000. The estimates for the market expected return and volatility are 0.1629 and 0.1837 respectively. The risk-free rate is the average daily 91-day T-bill rate for 2000, which is 0.06001. Other parameters: $\gamma = 4.0$, $T_1 = T_2 = 10$ years.
3. ADC Telecom is omitted since it is not rated.