Another look at: if p, then q

From example: if $\underbrace{\text{it is raining}}_{p}$, $\underbrace{\text{the street is wet}}_{q}$

we reasoned that the negation: $\neg(p \to q) = p \wedge \neg q$ —

that is: "p and not q"

Thus:

$$p \to q \equiv \sim(\sim(p \to q)) \equiv \sim(p \wedge \sim q) \overset{\text{(by de Morgan laws)}}{\equiv} \sim p \vee q$$

We also know that for 2 propositions joined by $\vee$ the compound statement is true when either is true — and false when both propositions false

Thus: $p \to q \equiv \sim p \vee q$ is true whenever $\sim p$ is true

or equivalently when p is false

Thus the last 2 lines of truth table for $p \to q$

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

Show using truth tables that

$$p \to q \equiv \underbrace{\sim q \to \sim p}_{}$$

contrapositive

p	q	$\neg p$	$\neg q$	$p \to q$	$q \to p$	$\sim q \to \sim p$	$\sim p \to \sim q$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	F	F
F	T	T	F	T	F	T	F
F	F	T	T	T	T	T	T

Similarly $\underbrace{q \to p}_{\text{converse}} \equiv \neg p \to \neg q$

AND

$\sim(q \to p) \equiv \sim p \to \sim q$

And $p \to q \not\equiv q \to p$

Not always so easy to distinguish in everyday language.

Equivalent propositions

Two propositions p and q are equivalent

$p \equiv q$ means p and q have same truth values

exercise: Given $p \to q$ and $\sim q \to \sim p$ construct truth table for

(1) $(p \to q) \to (\sim q \to \sim p)$

(2) $(\sim q \to \sim p) \to (p \to q)$

Quantifiers

There are expressions that with input become propositions / statements

example: $x^2 - 5x + 6 = 0$

Input can be thought of as a variable — when variable specified with precision, expression becomes a statement that is either true or false.

To specify variable: (1) the set of objects from which variable is taken is specified

(2) it is determined that

① all elements of set are to be considered OR

② if only some are to be considered.

If we consider: "all", use symbol $\forall$ = "for every"

: "some", use symbol $\exists$ = "there exists"

Thus with example — let set specified be $\mathbb{R}$

$\in$ = "in"

set of real numbers:

(1) $\forall x \in \mathbb{R},\ x^2 - 5x + 6 = 0$ $\equiv$ for every real number $x^2 - 5x + 6 = 0$

(2) $\exists x \in \mathbb{R} : x^2 - 5x + 6 = 0$ $\equiv$ there exists x in $\mathbb{R}$ such that $x^2 - 5x + 6 = 0$

Negations of statements involving quantifiers

example

(1) all roses in garden are red

$\forall$ rose r in garden, r is red

(2) there is a rose in garden that is red

$\exists$ rose r in garden, such that r is red

Negation of (1) ?

$\neg(\forall$ rose r in garden, r is red$)$

$\equiv$ $\exists$ rose r in garden : r is NOT red

Negation of (2) ?

$\neg(\exists$ rose r in garden : r is red$)$

$\equiv$ $\forall$ rose r in garden, r is NOT red

Moral of story?

Nested quantifiers?

Definition continuity of function at a point