# Solutions to First Assignment

## Section 1.1

**28.** a) Converse: If I stay home, then it will snow tonight. Contrapositive: If I do not stay at home, then it will not snow tonight. Inverse: If it does not snow tonight, then I will not stay home.

b) Converse: Whenever I go to the beach, it is a sunny summer day. Contrapositive: Whenever I do not go to the beach, it is not a sunny summer day. Inverse: Whenever it is not a sunny day, I do not go to the beach.

c) Converse: If I sleep until noon, then I stayed up late. Contrapositive: If I do not sleep until noon, then I did not stay up late. Inverse: If I don't stay up late, then I don't sleep until noon.

**32.** To construct the truth table for a compound proposition, we work from the inside out. In each case, we will show the intermediate steps. In part (d), for example, we first construct the truth tables for $p \wedge q$ and for $p \vee q$ and combine them to get the truth table for $(p \wedge q) \rightarrow (p \vee q)$. For parts (a) and (b) we have the following table (column three for part (a), column four for part (b)).

| $p$ | $\neg p$ | $p \rightarrow \neg p$ | $p \leftrightarrow \neg p$ |
|---|---|---|---|
| T | F | F | F |
| F | T | T | F |

For parts (c) and (d) we have the following table.

| $p$ | $q$ | $p \vee q$ | $p \wedge q$ | $p \oplus (p \vee q)$ | $(p \wedge q) \rightarrow (p \vee q)$ |
|---|---|---|---|---|---|
| T | T | T | T | F | T |
| T | F | T | F | F | T |
| F | T | T | F | T | T |
| F | F | F | F | F | T |

For part (e) we have the following table.

| $p$ | $q$ | $\neg p$ | $q \rightarrow \neg p$ | $p \leftrightarrow q$ | $(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$ |
|---|---|---|---|---|---|
| T | T | F | F | T | F |
| T | F | F | T | F | F |
| F | T | T | T | F | F |
| F | F | T | T | T | T |

For part (f) we have the following table.

## Section 1.3

**10.** We construct a truth table for each conditional statement and note that the relevant column contains only T's. For part **(a)** we have the following table.

| $p$ | $q$ | $\neg p$ | $p \vee q$ | $\neg p \wedge (p \vee q)$ | $[\neg p \wedge (p \vee q)] \to q$ |
|---|---|---|---|---|---|
| T | T | F | T | F | T |
| T | F | F | T | F | T |
| F | T | T | T | T | T |
| F | F | T | F | F | T |

For part **(b)** we have the following table. We omit the columns showing $p \to q$ and $q \to r$ so that the table will fit on the page.

| $p$ | $q$ | $r$ | $(p \to q) \to (q \to r)$ | $q \to r$ | $[(p \to q) \to (q \to r)] \to (p \to r)$ |
|---|---|---|---|---|---|
| T | T | T | T | T | T |
| T | T | F | F | T | T |
| T | F | T | T | T | F |
| T | F | F | F | F | T |
| F | T | T | T | T | T |
| F | T | F | F | T | F |
| F | F | T | T | T | F |
| F | F | F | T | T | T |

For part **(c)** we have the following table.

| $p$ | $q$ | $p \to q$ | $p \wedge (p \to q)$ | $[p \wedge (p \to q)] \to q$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | F | T |
| F | T | T | F | T |
| F | F | T | F | T |

**16.** The first of these propositions is true if and only if $p$ and $q$ have the same truth value. The second is true if and only if either $p$ and $q$ are both true, or $p$ and $q$ are both false. Clearly these two conditions are saying the same thing.

## Assignment 2

### Sectiion 2.2

**22.** First we show that every element of the left-hand side must be in the right-hand side as well. If $x \in A \cap (B \cap C)$, then $x$ must be in $A$ and also in $B \cap C$. Hence $x$ must be in $A$ and also in $B$ and in $C$. Since $x$ is in both $A$ and $B$, we conclude that $x \in A \cap B$. This, together with the fact that $x \in C$ tells us that $x \in (A \cap B) \cap C$, as desired. The argument in the other direction (if $x \in (A \cap B) \cap C$ then $x$ must be in $A \cap (B \cap C)$) is nearly identical.

**24.** First suppose $x$ is in the left-hand side. Then $x$ must be in $A$ but in neither $B$ nor $C$. Thus $x \in A - C$, but $x \notin B - C$, so $x$ is in the right-hand side. Next suppose that $x$ is in the right-hand side. Thus $x$ must be in $A - C$ and not in $B - C$. The first of these implies that $x \in A$ and $x \notin C$. But now it must also be the case that $x \notin B$, since otherwise we would have $x \in B - C$. Thus we have shown that $x$ is in $A$ but in neither $B$ nor $C$, which implies that $x$ is in the left-hand side.

**30.** **a)** We cannot conclude that $A = B$. For instance, if $A$ and $B$ are both subsets of $C$, then this equation will always hold, and $A$ need not equal $B$.

**b)** We cannot conclude that $A = B$; let $C = \emptyset$, for example.

**c)** By putting the two conditions together, we *can* now conclude that $A = B$. By symmetry, it suffices to prove that $A \subseteq B$. Suppose that $x \in A$. There are two cases. If $x \in C$, then $x \in A \cap C = B \cap C$, which forces $x \in B$. On the other hand, if $x \notin C$, then because $x \in A \cup C = B \cup C$, we must have $x \in B$.

**48.** We note that these sets are increasing, that is, $A_1 \subseteq A_2 \subseteq A_3 \subseteq \cdots$. Therefore, the union of any collection of these sets is just the one with the largest subscript, and the intersection is just the one with the smallest subscript.

**a)** $A_n = \{\ldots, -2, -1, 0, 1, \ldots, n\}$ **b)** $A_1 = \{\ldots, -2, -1, 0, 1\}$