Section 2.3 Problems: 22,30,42

22. If we can find an inverse, the function is a bijection. Otherwise we must explain why the function is not on-to-one or not onto.

a) This is a bijection since the inverse function is $f^{-1}(x) = (4 - x)/3$.

b) This is not one-to-one since $f(17) = f(-17)$, for instance. It is also not onto, since the range is the interval $(-\infty, 7]$. For example, 42548 is not in the range.

c) This function is a bijection, but not from **R** to **R**. To see that the domain and range are not **R**, note that $x = -2$ is not in the domain, and $x = 1$ is not in the range. On the other hand, f is a bijection from $\mathbf{R} - \{-2\}$ to $\mathbf{R} - \{1\}$, since its inverse is $f^{-1}(x) = (1 - 2x)/(x - 1)$.

d) It is clear that this continuous function is increasing throughout its entire domain ($\mathbf{R}$) and it takes on both arbitrarily large values and arbitrarily small (large negative) ones. So it is a bijection. Its inverse is clearly $f^{-1}(x) = \sqrt[5]{x - 1}$.

30. In all parts, we simply need to compute the values $f(-1)$, $f(0)$, $f(2)$, $f(4)$, and $f(7)$ and collect the values into a set.

a) $\{1\}$ (all five values are the same) **b)** $\{-1, 1, 5, 8, 15\}$ **c)** $\{0, 1, 2\}$ **d)** $\{0, 1, 5, 16\}$

42. a) The answer is the set of all solutions to $x^2 = 1$, namely $\{1, -1\}$.

b) In order for x^2 to be strictly between 0 and 1, we need x to be either strictly between 0 and 1 or strictly between -1 and 0. Therefore the answer is $\{\, x \mid -1 < x < 0 \vee 0 < x < 1 \,\}$.

c) In order for x^2 to be greater than 4, we need either $x > 2$ or $x < -2$. Therefore the answer is $\{\, x \mid x > 2 \vee x < -2 \,\}$.

Section 2.4 Problems 12 & 14

12. **a)** $-3a_{n-1}+4a_{n-2}=-3\cdot 0+4\cdot 0=0=a_n$ **b)** $-3a_{n-1}+4a_{n-2}=-3\cdot 1+4\cdot 1=1=a_n$

c) $-3a_{n-1}+4a_{n-2}=-3\cdot(-4)^{n-1}+4\cdot(-4)^{n-2}=(-4)^{n-2}((-3)(-4)+4)=(-4)^{n-2}\cdot 16=(-4)^{n-2}(-4)^2=(-4)^n=a_n$

d) $-3a_{n-1}+4a_{n-2}=-3\cdot(2(-4)^{n-1}+3)+4\cdot(2(-4)^{n-2}+3)=(-4)^{n-2}((-6)(-4)+4\cdot 2)-9+12=(-4)^{n-2}\cdot 32+3=(-4)^{n-2}(-4)^2\cdot 2+3=2\cdot(-4)^n+3=a_n$

14. In each case, one possible answer is just the equation as presented (it is a recurrence relation of degree 0). We will give an alternate answer.

a) One possible answer is $a_n=a_{n-1}$.

b) Note that $a_n-a_{n-1}=2n-(2n-2)=2$. Therefore we have $a_n=a_{n-1}+2$ as one possible answer.

c) Just as in part **(b)**, we have $a_n=a_{n-1}+2$.

d) Probably the simplest answer is $a_n=5a_{n-1}$.

e) Since $a_n-a_{n-1}=n^2-(n-1)^2=2n-1$, we have $a_n=a_{n-1}+2n-1$.

f) This is similar to part **(e)**. One answer is $a_n=a_{n-1}+2n$.

g) Note that $a_n-a_{n-1}=n+(-1)^n-(n-1)-(-1)^{n-1}=1+2(-1)^n$. Thus we have $a_n=a_{n-1}+1+2(-1)^n$.

h) $a_n=na_{n-1}$

Section 2.5 Problems 2 & 10

2. **a)** This set is countably infinite. The integers in the set are 11, 12, 13, 14, and so on. We can list these numbers in that order, thereby establishing the desired correspondence. In other words, the correspondence is given by $1\leftrightarrow 11$, $2\leftrightarrow 12$, $3\leftrightarrow 13$, and so on; in general $n\leftrightarrow(n+10)$.

b) This set is countably infinite. The integers in the set are -1, -3, -5, -7, and so on. We can list these numbers in that order, thereby establishing the desired correspondence. In other words, the correspondence is given by $1\leftrightarrow -1$, $2\leftrightarrow -3$, $3\leftrightarrow -5$, and so on; in general $n\leftrightarrow -(2n-1)$.

c) This set is $\{-999{,}999, -999{,}998, \ldots, -1, 0, 1, \ldots, 999{,}999\}$. It is finite, with cardinality 1,999,999.

d) This set is uncountable. We can prove it by the same diagonalization argument as was used to prove that the set of all reals is uncountable in Example 5.

e) This set is countable. We can list its elements in the order $(2,1),(3,1),(2,2),(3,2),(2,3),(3,3),\ldots$, giving us the one-to-one correspondence $1\leftrightarrow(2,1), 2\leftrightarrow(3,1), 3\leftrightarrow(2,2), 4\leftrightarrow(3,2), 5\leftrightarrow(2,3), 6\leftrightarrow(3,3),\ldots$.

f) This set is countable. The integers in the set are 0, ± 10, ± 20, ± 30, and so on. We can list these numbers in the order 0, 10, -10, 20, -20, 30, …, thereby establishing the desired correspondence. In other words, the correspondence is given by $1\leftrightarrow 0$, $2\leftrightarrow 10$, $3\leftrightarrow -10$, $4\leftrightarrow 20$, $5\leftrightarrow -20$, $6\leftrightarrow 30$, and so on.

10. In each case, let us take A to be the set of real numbers.

a) We can let B be the set of real numbers as well; then $A - B = \emptyset$, which is finite.

b) We can let B be the set of real numbers that are not positive integers; in symbols, $B = A - \mathbf{Z}^+$. Then $A - B = \mathbf{Z}^+$, which is countably infinite.

c) We can let B be the set of positive real numbers. Then $A - B$ is the set of negative real numbers and 0, which is certainly uncountable.

Section 4.1 Problem 14

14. This problem is equivalent to asking for the right-hand side **mod** 19. So we just do the arithmetic and compute the remainder upon division by 19.

a) $13 \cdot 11 = 143 \equiv 10 \pmod{19}$ **b)** $8 \cdot 3 = 24 \equiv 5 \pmod{19}$

c) $11 - 3 = 8 \pmod{19}$ **d)** $7 \cdot 11 + 3 \cdot 3 = 86 \equiv 10 \pmod{19}$

e) $2 \cdot 11^2 + 3 \cdot 3^2 = 269 \equiv 3 \pmod{19}$ **f)** $11^3 + 4 \cdot 3^3 = 1439 \equiv 14 \pmod{19}$