The Awakening of European Mathematics

Fibonacci, Tartaglia, Cardan

Fibonacci, 1170-1250

- European mathematics was not entirely dead in the Middle Ages. In fact the greatest mathematician anywhere during the Middle Ages was an Italian.
- His name was Leonardo of Pisa, but we have come to know him by a nickname, Fibonacci, which is a contraction of *filius Bonaccio,* meaning "son of Bonaccio."

Fibonacci, 2

- Born in Pisa and educated in North Africa, where his father was in charge of a customs house.
- He travelled widely in the Mediterranean area, noting the arithmetical systems of different countries.
- He quickly recognized the superiority of the Hindu-Arabic numeral system, with positional notation and a zero symbol, over Roman numerals which were still in use in Italy.

Fibonacci, *Liber Abaci*

- In 1202, he returned to Pisa and wrote his first book, *Liber Abaci (Book of Counting)*, which explained the advantages of the new number system “in order that the Latin race might no longer be deficient in that knowledge.”
- The first chapter begins:
  - These are the nine figures of the Indians:
    - 9 8 7 6 5 4 3 2 1
  - With these nine figures, and with this sign 0 … any number may be written, as will be demonstrated below.
- It was from the second edition of this book in 1228, that Europeans came to know and understand Arabic numerals.

Why do we call them Arabic numerals?

- Fibonacci clearly identifies the symbols as of Indian origin, and this is the work that convinced Europeans to switch over to this way of writing numbers.
- The reason we call them “Arabic” appears to be an outcome of Adelard of Bath’s translation of al-Khowârizimi’s arithmetic text, even though al-Khowârizimi identified them as Hindu.

European resistance to Arabic numerals

- Despite Fibonacci’s convincing book, Europeans did not jump to adopt the new way of writing numbers.
- In 1299, Florence issued an ordinance forbidding merchants from using Arabic numerals in bookkeeping. They either had to use Roman numerals or write out the numbers in words.
  - A possible reason is the ambiguity arising from the variety of ways that the numerals could be written, or the ease of altering some numerals to look like others, e.g., a zero could be changed into a 6 or a 9 by the addition of a tail.
  - Another possible reason is that people were not used to the concept of zero. (What does a symbol for nothing mean?)
  - Only after the invention of printing (around 1450) did the form of Arabic numerals become standardized – into essentially the form they still have today.
  - Outside of Italy accounts were kept in Roman numerals until about 1550, and for another humdred years in certain monasteries.

Some problems from the *Liber Abaci*

- Here is a problem posed by Fibonacci in the *Liber Abaci*:
  - Two birds start flying from the tops of two towers 50 feet apart; one tower is 30 feet high and the other 40 feet high. Starting at the same time and flying at the same rate, the birds reach a fountain between the bases of the towers at the same moment. How far is the fountain from each tower?
- How would you solve this?

Another problem from the *Liber Abaci*

- A merchant doing business in Lucca doubled his money there and then spent 12 denarii. On leaving, he went to Florence, where he also doubled his money and spent 12 denarii. Returning home to Pisa, he there doubled his money and again spent 12 denarii, nothing remaining.
- How much did he have in the beginning?

One more from *Liber Abaci*…

- Three men, each having some demarii, found a purse containing 23 denarii.
  - The first man said to the second, "If I take this purse, I will have twice as much as you."
  - The second said to the third, "If I take this purse, I will have three times as much as you."
  - The third man said to the first, "If I take this purse, I will have four times as much as you."
- How many denarii did each man have?

The Fibonacci Sequence

- Fibonacci is remembered today primarily as the originator of the Fibonacci Sequence.
- The name was given to this sequence by a 19th century French mathematician named Edouard Lucas who wrote about it. Lucas pointed out that the sequence was first described by Fibonacci in the *Liber Abaci* in a problem as follows:
  - A man put one pair of rabbits in a certain place entirely surrounded by a wall.
  - How many pairs of rabbits can be produced from that pair in a year, if the nature of these rabbits is such that every month each pair bears a new pair which from the second month onward becomes productive?

The Rabbit Colony

Months	Adult Pairs	Young Pairs	Total Pairs
1	1	1	2
2	2	1	3
3	3	2	5
4	5	3	8
5	8	5	13
6	13	8	21
7	21	13	34
8	34	21	55
9	55	34	89
10	89	55	144
11	144	89	233
12	233	144	377

The Fibonacci sequence defined

- The sequence, looking either at the adult pairs, or the young pairs, or the total number of pairs runs as follows:
  - 1,1,2,3,5,8,13,21,34,55,89,144,233,377,…
- This is the Fibonacci sequence. It can be defined as follows:
  - F_1 and F_2 each = 1
  - From there on, $F_n = F_{n-2} + F_{n-1}$, for $n \geq 3$.
    - In other words, the first two terms are 1, and from then on each term is the sum of the two preceding.

A Recursive Sequence

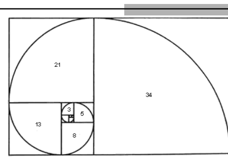
- A recursive sequence is one in which, beyond a certain point, every succeeding term is a linear combination of preceding terms.
- The Fibonacci sequence is perhaps the earliest known recursive sequence to be studied.

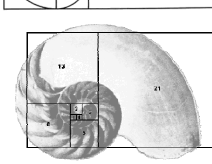
Properties of the Fibonacci sequence

- Definition: The terms of the Fibonacci sequence are called “Fibonacci numbers.”
- Edouard Lucas pointed out that the sum of the first *n* Fibonacci numbers equals one less than Fibonacci number *n+2.*
  - E.g., 1 + 1 + 2 + 3 + 5 + 8 + 13 + 21 = 55 – 1
- Another property: The square of any Fibonacci number (after the first two) is equal to the product of the previous Fibonacci number and the next Fibonacci number plus 1 if the the Fibonacci number is in an even number position or minus 1 if the number is in an odd position, i.e.:
  - $(F_n)^2 = F_{n-1}F_{n+1} + (-1)^{n-1}$, n≥1

Fibonacci sequences in nature

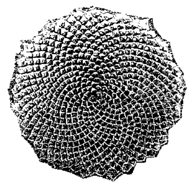
- Construct a sequence of squares with sides equal to the Fibonacci numbers, 1, 1, 2, 3, 5, 8, 13, …
- Place them next to each other to form a clockwise spiral. Draw a smooth curved line through diagonally opposite corners of the squares, as in the upper diagram. The result will be a growing spiral as shown.
- In fact it will also describe the structure of the chambers of the shell of a *nautilus,* an oceanic cephalopod. The structure gives the nautilus balance and strength.





Fibonacci sequences in nature, 2

- The Fibonacci sequence also describes the arrangement of the spiral of florets in the center of a sunflower.
- And it appears again and again in Nature and in apparently man-made numbers, such as patterns of stock market prices.



The influence of the *Liber Abacci*

- The *Liber Abacci* was the work most responsible for moving Europe away from Roman numerals and adopting the Arabic numeral system, with place values, zeros, and clear methods for doing arithmetic.
- It also introduced Europe to many algebraic procedures and ways to solve difficult problems.
- Despite this, it circulated only in hand-written copies. It was not printed (at least in Italy) until 1857, nor translated into English.

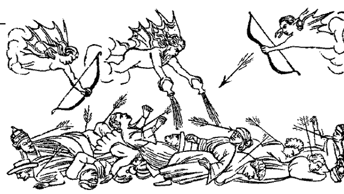
After Fibonacci

- Fibonacci died in 1250.
- Europe was in the process of digesting the great mass of scholarship that they had discovered in the Arab world, thanks to scholars like Fibonacci who were familiar with it at first hand and then through the Great Translation Project, after the Crusades.
- Feudalism was producing a higher standard of living. Life was improving all around.
- Europe was getting back on its feet after the collapse of the Roman Empire a thousand years before.
- And then…

The Black Death

- Bubonic plague hit Europe in **1346**.
- The Black Death eliminated 1/3 to 1/2 of population of Europe.

The Black Death as Retribution



- This illness was called the Black Death because sores, skin, blood, vomit, etc., all turned black before the patient died, often within a few hours of becoming ill.
- At the time it was widely thought to be divine retribution for sinful living. It was worst in port cities.

A New Beginning for Europe

- After the Black Death, the economy of Europe took off.
- Fewer people sharing the same resources.
- Emphasis on labour-saving devices.
- The Renaissance began in earnest.

The Demand for Books

- When the Renaissance got underway, more and more people in Europe wanted access to the mass of written scholarship that was accumulating from the translations of Moslem works from Arabic and of Byzantine works from Greek.
- Paper from Linen was readily available.
- But scribes were in great demand and could not keep up.

Printed Block Books

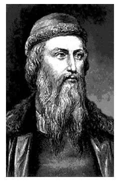
- A means of printing any written material mechanically would be in demand.
- Mechanical printing was known in China for at least 1000 years.
- But the method required making a woodcut of an entire page (in mirror image), then inking the woodcut, then applying it to a piece of paper.

Printing from Movable Type

- Some “block books” were printed in Europe in the 15th century.
- But the process was cumbersome, and error-prone.
  - A single mistake on one page required that the entire page be re-carved.
- A method was needed to assemble a page, letter by letter.

Gutenberg’s Printing Press

- Johann Gutenberg (1400-1468).
  - Goldsmith from Mainz, Germany.
  - Found a method of manufacturing individual letter slugs (that print the letter) of a uniform height so they could be lined up on a bed, inked, and a sheet of paper pressed against them to print a page.



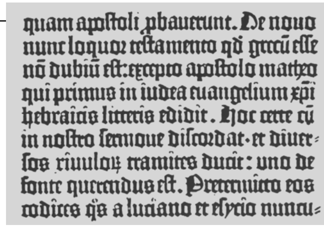
Gutenberg’s matrix

- Gutenberg’s trick was to use the same mould for all letter slugs, but placing a different letter impression on the end to make different letters.
- A molten lead alloy is poured into a matrix with the letter impression on the end, held together with a firm spring.
- The matrix adjusts to the width of the letters desired.
  - Wide for “M”, narrow for “I”, etc.

The Bed of the Press

- The desired letters for a page are then arranged and lined up on a flat bed, secured in place and inked.
- A sheet of paper is loaded into a frame held above the bed and then pressed onto the inked letters with a forceful screw press.

The finished product


quam apostoli pbauerunt. De nouo
nunc loquor testamento qd grecu esse
no dubiu est: excepto apostolo matheo
qui primus in iudea euangelium xpi
hebraicis litteris edidit. Hoc certe cu
in nostro sermone discordat. et diuer-
sos riuulos tramites ducit: uno de
fonte querendus est. Pretermitto eos
codices qs a luciano et esycio nuncu-

- The printed page can contain any text whatsoever that can be written in Latin letters.
- When sufficient copies are made, the letter slugs are removed and sorted ready for another page to be composed.

A 16th century print shop

- Printing was immensely successful. In the 50 years from 1450 to 1500 more than 10 million volumes were printed.

Printing changed the world

- The character of education, general knowledge, the dissemination of information, and all the infrastructure of civilization changed dramatically with the invention of printing.

The Printing of Mathematical works

- The first printed books had little to do with mathematics.
- Mostly they were religious texts, funded by the Church. The Bible itself was the most printed book by almost all printers.
- The Humanism movement in Italy, which predated mechanical printing, was mostly interested in the classical works of philosophy and literature. Mathematics and science were associated with the Arab world and were often shunned.
- Also, with mathematics, there was the problem of reproducing symbols and diagrams.

The first printed mathematics works

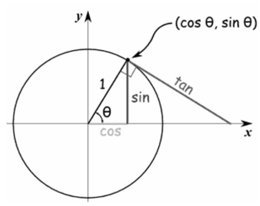
- The first works of mathematics printed were not very profound.
  - E.g., *Treviso Arithmetic*, published in 1478 in Treviso, Italy, a mercantile town north of Venice, was a popular textbook. Essentially it was a list of rules for making common calculations, intended for commerce.
  - But it began a movement of publishing mathematical texts in Italy.
- In 1482, a Latin translation of Euclid was published in Venice – a poor translation from the Arabic version.
- In 1505, a better translation was made directly from a recovered Greek manuscript.

De Triangulis

- Johannes Müller (1436-1476), better known as Regiomontanus, was a mathematician and astronomer from Königsberg, Germany.
- He translated many classical mathematical works, including Ptolemy's *Almagest*.
- His greatest original work was *De Triangulis Omnimodis* (On Triangles of all Kinds), finished in 1464, but not printed until 1533.
- This work introduced Europe to trigonometry.

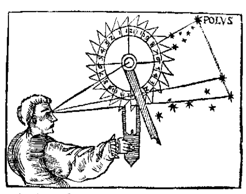
Trigonometry

- Regiomontanus' work had the effect of separating trigonometry as a separate branch of mathematics, independent of astronomy.
  - In ancient times, the Table of Chords, used by Ptolemy to compute stellar distances was among the main uses of trigonometric concepts.
- Only the sine and cosine function were presented in *De Triangulis*. Later Regiomontanus computed a table of tangents.



Influence of trigonometry

- Trigonometry became the essential mathematics for the great voyages of discovery. Applying trigonometric principles to finding one's position on Earth, not just the relative positions of heavenly bodies, emboldened explorers to venture to unknown parts of the world.
- Columbus was known to have carried one of Regiomontanus' works with him on all four of his trips to the New World.



A quadrant, for determining latitude from a sighting of the pole star.

Mathematical Encyclopedias

- Renaissance scholars often attempted to sum up everything that was known about some subject in a single, comprehensive work.
- Luca Pacioli (1445-1514), commonly called Fra Luca di Borga, published a large compendium, *Summa de Arithmetica Geometria Proportioni et Proportionalita,* in 1494 in Venice, which proved to be the most influential mathematical book of that era. It contained almost nothing that was not in Fibonacci's *Liber Abaci* from 300 years earlier, showing how little mathematics had progressed in that period. (But Fibonacci's work had not been printed.)
  - It also introduced and explained double-entry bookkeeping.

The Founding of the Universities

- The original *universitas* was merely a collection of individuals who came together to communicate ideas.
- The original *universitas* were located at the monastery and cathedral schools founded by Charlemagne, and their purpose was to prepare students for the priesthood.
- Their function was to preserve, not advance, knowledge.
- The reputation of a particular school depended largely on the reputation of its teachers. Teachers were paid directly by the students. A famous teacher attracted large numbers of students and enriched the town where the school was located.

Student organization

- Students banded together to protect themselves from extortion by local citizens and to secure legal rights. These associations formed the administrative structure of the emerging universities.
- Eventually they gained legal recognition through a charter from the king or the pope.
  - All the early medieval universities were formed this way: Bologna (1158), Paris (1200), Padua (1222), Oxford (1214), and Cambridge (1231).

The Seven Liberal Arts

- As at Charlemagne's cathedral schools, the curriculum at the new universities followed the seven liberal arts of Martianus Capella:
  - The Trivium
    - Grammar, Dialectic, Rhetoric
  - The Quadrivium
    - Geometry, Arithmetic, Astronomy, Music
- This would suggest that mathematics was given a high place of importance, but in fact little attention was given to the Quadrivium on the grounds that those subjects were "practical" and a university education was held to be "general."

Forcing a little mathematical knowledge

- Neo-Platonic influences led university faculties to decide that some mathematics was a good thing. Hence it began to be made a requirement:
  - At the University of Paris in 1336 a statute was passed that no student could graduate without attending lectures on "some mathematical books." After 1452, candidates for the Master of Arts had to take an oath that they had read the first 6 books of Euclid!

Mathematical contests

- It was typical of the Renaissance that people known as mathematicians were self-proclaimed clever people who could solve difficult mathematical problems put to them in contests.
- Mathematicians would challenge each other to public contests where each would pose problems to the other to solve, to show who was the cleverer.
- On the basis of the outcome of these contests, great sums of money would pass hands, and academic appointments hung on one's result in these "jousts."

Mathematical secrecy

- Unlike today, when mathematicians gain their reputations by publishing their mathematical discoveries in widely available journals, in the Renaissance, a mathematician who made a discovery of how to solve certain kinds of problems would keep the solution secret in order to use the problems in public contests.

The Cubic Equation

- One of the most important problems of algebra that had not been solved by any of those who wrote about algebra in ancient or medieval times was that of solving cubic equations.
- In general, a cubic equation is one of the form:
  - $ax^3 + bx^2 + cx + d = 0$

Solution of the cubic viewed as impossible

- It was widely thought in the Renaissance that the solution of the cubic was as impossible as that of squaring the circle.
- Fra Luca Pacioli ended his book, the *Summa*, with the assertion that it was impossible.
- Pacioli had lectured at the University of Bologna in 1501-02. One of his colleagues there was Scipione del Ferro (1465-1526), who took the challenge and found a solution to the special case of the cubic with the form $x^3 + px = q$, where p and q are positive.

Keeping the solution secret

- del Ferro kept his solution secret so that he could use it in public contests.
- At the end of his life he divulged the solution to his pupil and successor at Bologna, Antonio Maria Fiore.
- Fiore then used this knowledge to challenge one of the prominent mathematicians of his time to a contest.
- Out of this came one of the nastiest priority disputes in mathematics…

Niccolo Tartaglia (1500-1557)

- The mathematician that del Ferro challenged was Niccolo Tartaglia.
- His real name was Niccolo Fontana. He was badly injured in as a boy by a sabre that cut his jaw and palate and nearly died. His wound healed but left him with a bad speech defect, from which he got his nickname, which stuck:
  - "Tartaglia" means "the stammerer." Later in life he used this nickname in his published works.

Tartaglia

- Tartaglia was born into poverty, but managed to educate himself, especially in mathematics.
- He began to earn a living by teaching mathematics in Verona and Venice and developed a reputation.
- In 1530 he was sent two problems by a friend that involved cubic equations.
  - It took him five years, but eventually he solved both problems. When he did so, he announced to the world that he could solve any problem of the form $x^3 + px^2 = q$.

The Fiore-Tartaglia contest

- del Fiore believed that Tartaglia was bluffing, so he challenged him to a public problem-solving contest. Each man was to propose 30 problems. The victor would be the one who could solve the greatest number within 50 days.
- Tartaglia knew that del Fiore had learned of the solution of some form of the cubic from a deceased master, so he worked feverishly to find the general solution.
  - Not long before the contest was to begin, Tartaglia found a scheme to solve all cubics that lacked the second-degree term, i.e. those of the form $ax^3 + 0x^2 + cx + d = 0$.
  - Thus, before the contest began, Tartaglia, who still lacked a general solution, could solve two special cases, those lacking the second-degree and those lacking the first-degree.
  - Fiore only knew how to handle one type.
- When the contest began, within two hours, Tartaglia had reduced all 30 problems posed by Fiore to particular cases of $x^3 + px = q$, which he could answer. Fiore, on the other hand, could not solve any of the problems that Tartaglia put to him.

Enter Cardano

- Girolamo Cardano, about whom more later, heard about the contest where he was in Milan. He begged Tartaglia for the solution to the cubic equation. He said he would publish it in his forthcoming book, credited to Tartaglia.
- Tartaglia refused, saying he was going to publish a book on algebra himself, where he would announce the solution.
- Cardan persisted and after much flattery and more begging, Tartaglia divulged the solution to Cardan, but swore him to secrecy.

Cardan wriggles out of the oath

- Tartaglia had a reputation for claiming he was the originator of ideas which he had in fact copied from others.
- Cardan suspected that the cubic solution was not discovered by Tartaglia. So he travelled to Bologna where he learned that del Ferro had some kind of solution to the cubic long before.
- Cardan convinced himself that Tartaglia was not the originator of the solution and therefore his oath to him was not binding.

Cardan publishes the solution

- In 1545, Cardan published his work *Ars Magna* in which he published the full solution and proof.
- He credited Tartaglia with the solution of the special cubic $x^3 + px = q$, but claimed that he worked out the proof himself.
- Tartaglia was livid and accused Cardan of lying. Their feud continued the rest of their lives leading to the worst case of ill-will among mathematicians known to that time.

Girolamo Cardano (1501-1576), a true Renaissance man

- One of the most colourful figures in the history of mathematics, Cardano, known in English as Jerome Cardan, had by all accounts a miserable life.
- One of his sons was executed for poisoning his wife. Cardan himself chopped off the ears of a second son who was trying to poison *his* wife.
- Cardan was imprisoned for heresy after casting a horoscope of Christ.
- In his autobiography, Cardan claims that his mother tried to induce an abortion when she was pregnant with him, but failed.

The Life of Girolamo Cardano

- Girolamo Cardano, aka Hieronimo Cardano; in Latin, Hieronymus Cardanus; in English, Jerome Cardan.
- Cardan was the illegitimate son of Fazio Cardano and Chiara Micheria.
  - Fazio was a lawyer in Milan, but considered an expert in mathematics. He lectured on geometry at the University of Pavia and at the Piatti foundation in Milan.
  - Chiara Micheria was a young widow in her 30s with 3 young children when she met Fazio, then in his 50s. When Chiara was pregnant with Girolamo, the plague hit Milan. She fled Milan for nearby Pavia to stay with friends of Fazio. Girolamo was born in Pavia, but meanwhile Chiara's first 3 children, who had been left behind in Milan, died of the plague.
    - Chiara lived apart from Fazio for many years, but eventually they did marry.

The Life of Girolamo Cardano, 2

- As a youth, Cardan worked as his father's assistant, but wished for a career in academe. Fazio had taught mathematics to Cardan.
- Cardan entered the University of Pavia as a medical student – though his father wanted him to study law.
- War between Spain and France (which then controlled Milan) forced the closure of the University of Pavia, while Cardan was a student there. As a result Cardan moved to the University of Padua to complete his medical studies.
  - Before completing his studies, Cardan campaigned successfully to become the Rector of the University of Padua. Becoming Rector entitled Cardan to receive the Doctor of Medicine degree immediately.

The Life of Girolamo Cardano, 3

- Cardan's father died while he was still a student. Cardan quickly squandered the small inheritance he received from his father, so he turned to gambling to make money. He engaged in card games, chess, and dice games and succeeded in winning more than he lost. He became addicted to gambling.
- Cardan, doctorate in hand, wished to move back to Milan, where his mother still lived, and practice medicine. The College of Physicians in Milan did not wish to admit him (though he was considered brilliant) because of his unpleasant personality. He eventually was refused because of his illigetimate birth.

The Life of Girolamo Cardano, 4

- Instead, Cardano set up his medical practice in Sacco, a small village near Padua.
- There, in 1531, he maried Lucia Bandarini, a local resident. But his medical practice did not make enough income to support a wife, so in 1532, he moved to a village near Milan.
- Again refused admittance to the Milan College of Physicians, Cardan reverted to gambling to make a living.
  - Alas, the gambling did not pay this time. Cardan pawned his wife's jewellery and sold their furniture.
  - They ended up in the poorhouse.

The Life of Girolamo Cardano, 5

- Things look up for Cardan:
  - Cardan obtained his father's former post as lecturer in mathematics at the Piatti Foundation in Milan.
  - He also practiced medicine (illegally) in Milan, and gained a reputation as a successful doctor. Eventually the College of Physicians changed its rules about illegitimate births and admitted Cardan in 1539.
  - In the same year, Cardan published two books on mathematics based on his lectures at the Piatti Foundation. These proved to be very popular.
  - Cardan began a prolific career as a writer, publishing books on medicine, philosophy, astronomy, theology, and, of course, mathematics.

The Life of Girolamo Cardano, 6

- In 1545, Cardan published his greatest mathematical work, the *Ars Magna*, which presented the solution to the cubic equation that he had extracted from Tartaglia, along with many other advances in algebra.
- Among the topics covered in *Ars Magna*, he discusses the solution of problems that involve imaginary numbers (square roots of negative numbers), though without really understanding quite what he was doing.
- His reputation as a physician grew and he was elected Rector of the College of Physicians in Milan. He gained the reputation of being the greatest physician in Europe and was sought after by many heads of state.
  - He travelled to Scotland to treat the Archbishop of St. Andrews, who had offered him a huge sum to cure him. Apparently he succeeded and was paid 2000 gold crowns.
  - He returned to Italy and was appointed professor of medicine at Pavia University and grew rich as a writer and physician.

The Life of Girolamo Cardano, 7

- Misfortune strikes:
  - In 1557, Cardan's eldest son qualified as a doctor and married a woman whose parents began a concerted attempt to extract as much money from their daughter's famous father in law as possible.
  - Cardan's son in frustration poisoned his wife and was arrested for murder and executed.
  - Cardan became a hated man as a father of a convicted murderer.
  - Cardan's younger son became a gambler and gambled away all of his possessions and a lot of his father's money. Cardan had him banished.
  - In 1570, Cardan was jailed for heresy for casting a horoscope of Jesus. He was released after a few months but barred from holding a university post or publishing more books.

The Life of Girolamo Cardano, 8

- Amazingly, on his release, Cardan went to Rome and was immediately granted membership in the College of Physicians there.
- And, the Pope granted him a pension.
- Cardan also cast his own horoscope in which he predicted the date of his own death.
  - It is claimed that in order to assure its accuracy, he committed suicide when the day arrived.

Cardan's Output

- At Cardan's death, he had published 131 books, 111 remained unpublished in manuscript, and he claimed to have burned another 170 which were unsatisfactory.
- Some of these were just re-workings of the writings of others, but many had groundbreaking insights.
- The *Ars Magna*, contained much new on solving algebraic problems.
- One of those unpublished at his death and eventually published in 1663 was *Liber de Ludo Aleae* (Book on Games of Chance), which was really the first book on the theory of probability, 50 years before the original work by Pascal and Fermat, who are usually credited with beginning the subject of probability.