Tycho Brahe and Johannes Kepler

The Music of the Spheres

Tycho Brahe

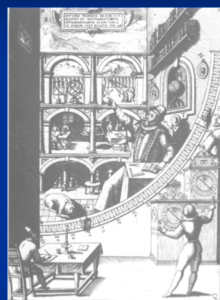
- 1546-1601
- Motivated by astronomy's predictive powers.
- Saw and reported the *Nova* of 1572.
- Considered poor observational data to be the chief problem with astronomy.

Tycho Brahe at Uraniborg

- Established an observatory--*Uraniborg* on Hven, an island off Denmark.
  - Worked there 20 years.
  - Became very unpopular with the local residents.

Tycho's Observations

- Made amazingly precise observations of the heavens with naked-eye instruments.
- Produced a huge globe of the celestial sphere with the stars he had identified marked on it.

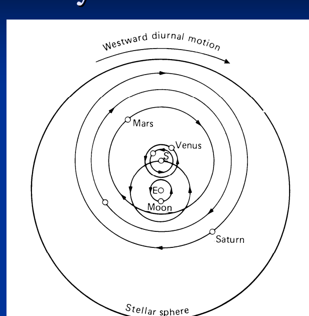


Tycho, the Imperial Mathematician

- Left Uraniborg to become the Imperial Mathematician to the Holy Roman Emperor at the Court in Prague.
- Tycho believed that Copernicus was correct that the planets circled the Sun, but could not accept that the Earth was a planet, nor that it moved from the centre of the universe.
- He developed his own compromise system.

Tycho's System

- Earth stationary.
- Planets circle Sun.
- Sun circles Earth.
- Problem:
  - Could not get *Mars* to fit the system.
  - Note the intersecting paths of the Sun and Mars that bothered Copernicus.



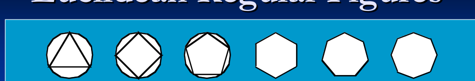
Johannes Kepler

- 1571-1630
- Lutheran
- Mathematics professor in Austria (Graz)
- Sometime astrologer
- Pythagorean/Neo-Platonist
- One of the few Copernican converts

Pythagorean/Platonic regularities in the Heavens

- Why are there precisely 6 planets in the heavens (in the Copernican system)?
  - Mercury, Venus, Earth, Mars, Jupiter, Saturn
  - With a Pythagorean mindset, Kepler was sure there was some mathematically necessary reason.
  - He found a compelling reason in Euclid.
    - A curious result in solid geometry that was attributed to Plato.

Euclidean Regular Figures

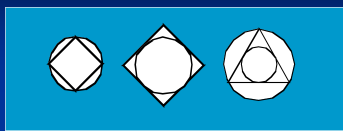


A regular figure is a closed linear figure with every side and every angle equal to each other.

- For example, an equilateral triangle, a square, an equilateral pentagon, hexagon, and so forth.

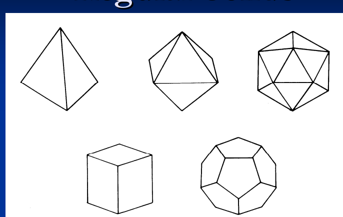
There is no limit to the number of regular figures with different numbers of sides.

Inscribing and Circumscribing



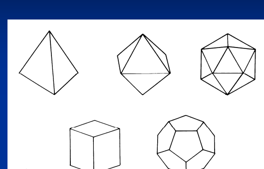
- All regular figures can be inscribed within a circle and also circumscribed around a circle.
- The size of the figure precisely determines the size of the circle that circumscribes it and the circle that is inscribed within it.

Regular Solids



- In three dimensions, the comparable constructions are called regular solids.
- They can inscribe and be circumscribed by spheres.

The Platonic Solids



- Unlike regular figures, their number is not unlimited. There are actually only five possibilities:
  - Tetrahedron, Cube, Octahedron, Dodecahedron, Icosahedron
- This was discussed by Plato. They are traditionally called the "Platonic Solids."
- That there could only be five of them was proved by Euclid in the last proposition of the last book of *The Elements*.

Kepler's brainstorm

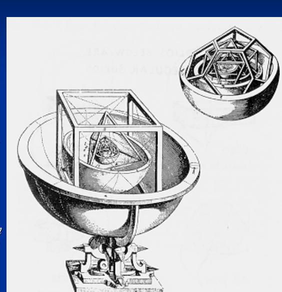
- Kepler imagined that (like Eudoxean spheres), the planets were visible dots located on the surface of nested spherical shells all centered on the Earth.
- There were six planets, requiring six spherical shells. Just the number to be inscribed in and circumscribe the five regular solids.

Six Planets, Five Solids

- Like Pythagoras, Kepler believed that neat mathematical relationships such as this could not be a coincidence. It must be the key to understanding the mystery of the planets.
- There were six planets because there were five Platonic solids. The "spheres" of the planets were separated by the inscribed solids. Thus their placement in the heavens is also determined.

The Cosmographical Mystery

- In 1596, Kepler published a short book, *Mysterium Cosmographicum*, in which he expounded his theory.
- The 6 planets were separated by the 5 regular solids, as follows:
  - Saturn / *cube* / Jupiter / *tetrahedron* / Mars / *dodecahedron* / Earth / *icosahedron* / Venus / *octahedron* / Mercury



What was the mystery?

- The cosmographical mystery that Kepler "solved" with the Platonic solids was the provision of reasons why something that appeared arbitrary in the heavens followed some exact rule. This is classic "saving the appearances" in Plato's sense.
- The arbitrary phenomena were:
  - The number of planets.
  - Their spacing apart from each other.
- Both were determined by his arrangement of spheres and solids.

Kepler and Tycho Brahe

- Kepler's cosmic solution didn't exactly work, but he thought it would with better data.
- Tycho had the data.
- Meanwhile Tycho needed someone to do calculations for him to prove his system.
- A meeting was arranged between the two of them.

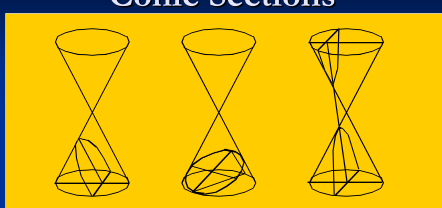
Kepler, the Imperial Mathematician

- Kepler became Tycho's assistant in 1600.
- Tycho died in 1601.
- Kepler succeeded Tycho as Imperial Mathematician to the Holy Roman Emperor in Prague, getting all of Tycho's data.

Kepler's "Discoveries"

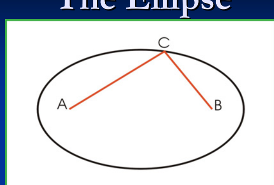
- Kepler found many magical and mysterious mathematical relations in the stars and planets.
- He published his findings in two more books:
  - *The New Astronomy*, 1609
  - *The Harmony of the World*, 1619
- Out of all of this, three laws survive.
  - The first two involve a new shape for astronomy, the *ellipse*.

Conic Sections



In addition to Euclid, Kepler would have known of the work of the Hellenistic mathematician Apollonius of Perga, who wrote a definitive work on what are called conic sections: the intersection of a cone with a plane in different orientations. Above are the sections Parabola, Ellipse, and Hyperbola.

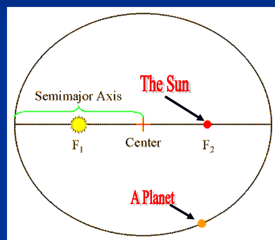
The Ellipse



- The Ellipse is formed by a plane cutting completely through the cone.
- Another way to make an ellipse is with two focal points (A and B above), and a length of, say, string, longer than the distance AB. If the string is stretched taut with a pencil and pulled around the points, the path of the pencil point is an ellipse. In the diagram above, that means that if C is any point on the ellipse, AC+BC is always the same.

Kepler's first law

- 1. The planets travel in elliptical orbits with the sun at one focus.
  - All previous astronomical theories had the planets travelling in circles, or combinations of circles.
  - Kepler has chosen a different geometric figure.



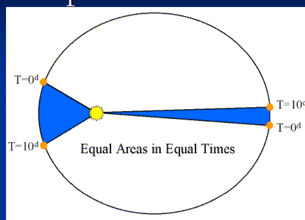
A radical idea, to depart from circles

Kepler's ideas were very different and unfamiliar to astronomers of his day.

What was the mystery?

- Kepler's "first law" gives some account of the actual paths of the planets (i.e., "saves" them).
- All of the serious astronomers before him had found that simple circular paths didn't quite work. Ptolemy's Earth-centered system had resorted to arbitrary epicycles and deferents, often off-centre. Copernicus also could not get circles to work around the sun.
- Kepler found a simple geometric figure that described the path of the planets around the sun.

Kepler's second law



- 2. A radius vector from the sun to a planet sweeps out equal areas in equal times.

What is the mystery here?

- The second law provides a mathematical relationship that accounts for the apparent speeding up of the planets as they get nearer the sun and slowing down as they get farther away.
- Kepler had no explanation *why* a planet should speed up near the sun. (He speculated that the sun gave it some encouragement, but didn't know why.)
- But in Platonic fashion he provided a formula that specifies the relative speeds.

Kepler's third law

- 3. The Harmonic Law: $d^3/t^2 = \mathrm{k}$
  - The cube of a planet's mean distance *d* from the sun divided by the square of its time *t* of revolution is the same for all planets.
  - That is, the above ratio is equal to the same constant, k, for all planets.

The mystery cleared up by the third law

- Kepler noted that the planets all take different times to complete a full orbit of the Sun.
- The farther out a planet was from the Sun, the longer was its period of revolution.
- He wanted to find a single unifying law that would account for these differing times.
- The 3rd law gives a single formula that relates the periods and distances of all the planets.
  - As usual, Kepler did not provide a cause for this relationship.

Kepler's three laws at a glance

- 1. The planets travel in elliptical orbits with the sun at one focus.
  - Accounts for the orbital paths of the planets.
- 2. A radius vector from the sun to a planet sweeps out equal areas in equal times.
  - Accounts for the speeding up and slowing down of the planets in their orbits.
- 3. The Harmonic Law: $d^3/t^2 = \mathrm{k}$
  - Accounts for the relative periods of revolution of the planets, related to their distances from the sun.

Why these are called Kepler's laws

- Kepler did not identify these three statements about the behaviour of the planets as his "laws."
  - We call these Kepler's laws because Isaac Newton pulled them out of Kepler's works and gave Kepler credit for them.
- Kepler found many "laws"—meaning regularities about the heavens—beginning with the cosmographical mystery and the 5 Platonic solids.
  - Most of these we ignore as either coincidences or error on his part.

What did Kepler think he was doing?

- Kepler has all the earmarks of a Pythagorean.
  - A full and complete explanation is nothing more nor less than a mathematical relationship describing the phenomena.
  - In Aristotle's sense it is a formal cause, but not an efficient, nor a final cause.

The Music of the Spheres

- As a final example of Kepler's frame of mind, consider the main issue of his last book, *The Harmony of the World.*
- Kepler's goal was to explain the harmonious structure of the universe.
- By harmony he meant the same as is meant in music.

Music of the Spheres, 2

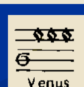
- Since Pythagoras it has been known that a musical interval has a precise mathematical relationship. Hence all mathematical relations, conversely, are musical intervals.
- If the planets' motions can be described by mathematical formula, the planets are then performing music.

Music of the Spheres, 3

- In particular, the orbits of the planets, as they move through their elliptical paths, create different ratios, which can be expressed as musical intervals.
- The angular speeds at which the planets move determine a pitch, which rises and falls through the orbit.

Music of the Spheres, 4

- As follows:
- Mercury, a scale running a tenth from C to E
- Venus—almost a perfect circular orbit—sounds the same note, E, through its orbit.
- Earth, also nearly circular, varies only from G to A-flat



Music of the Spheres, 5

- Mars, which has a more irregular path than Venus or the Earth, goes from F to C and back.
- Jupiter moves a mere minor third from G to B-flat.
- Saturn moves a major third from G to B.
- The Moon too plays a tune, from G to C and back.