Lecture 13

Inferential statistics:

Hypothesis testing

General steps for conducting hypothesis testing

Reading materials: Chapter 8 (or 9 website leanring center) of text book

Hypothesis testing is a decision-making process for evaluating claims about a population

The three methods used to test hypotheses are

1. The traditional method
2. The P-value method
3. The confidence interval method

Hypothesis Testing

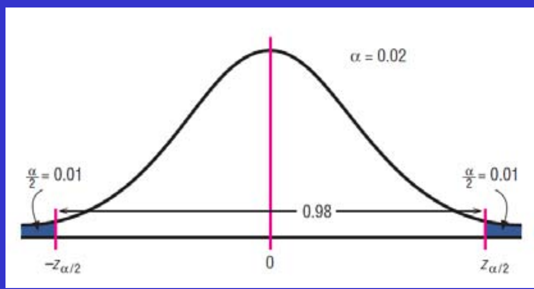
The **null hypothesis,** symbolized by H_0, is a statistical hypothesis that states that there is no difference between a parameter and a specific value, or that there is no difference between two parameters.

The **alternative hypothesis,** symbolized by H_1, is a statistical hypothesis that states the existence of a difference between a parameter and a specific value, or states that there is a difference between two parameters.

The null and alternative hypotheses are stated together, and the null hypothesis contains the equal sign, as shown (where *k* represents a specified number).

Two-tailed test	Right-tailed test	Left-tailed test
H_0: $\mu = k$	H_0: $\mu \le k$	H_0: $\mu \ge k$
H_1: $\mu \ne k$	H_1: $\mu > k$	H_1: $\mu < k$

Find $z_{\alpha/2}$ value



The relationship between α and the confidence level is that the stated confidence level is the percentage equivalent to the decimal value of $1 - \alpha$, and vice versa. When the 95% confidence interval is to be found, $\alpha = 0.05$, since $1 - 0.05 = 0.95$, or 95%. When $\alpha = 0.01$, then $1 - \alpha = 1 - 0.01 = 0.99$, and the 99% confidence interval is being calculated.

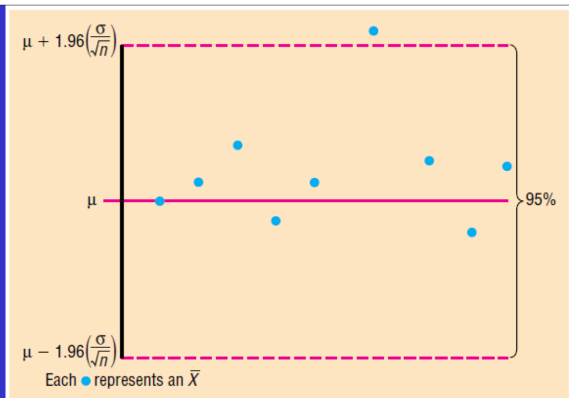
Formula for the Confidence Interval of the Mean for a Specific α

$$\bar{X} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{X} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

For a 95% confidence interval, $z_{\alpha/2} = 1.96$; and for a 99% confidence interval, $z_{\alpha/2} = 2.58$.

$$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

$$(-z_{a/2} < z < z_{a/2})$$

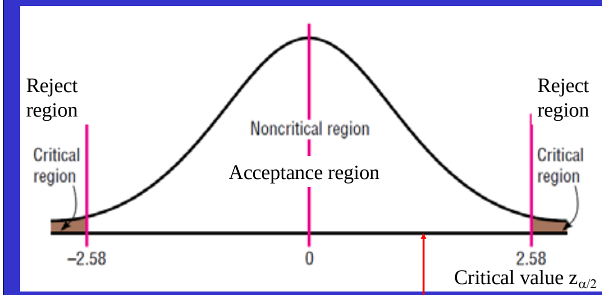


Approximately 95% of the sample means fall within 1.96 standard deviations of the population mean

A **statistical test** uses the data obtained from a sample to make a decision about whether or not the null hypothesis should be rejected.

The numerical value obtained from a statistical test is called the **test value.**

Statistic parameter



	H_0 True	H_0 False
Reject H_0	**Error** Type I	Correct decision
Do not reject H_0	Correct decision	**Error** Type II

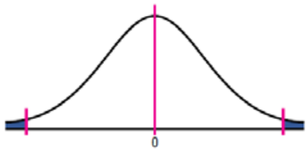
Reject or accept H_0

A **type I error** occurs if one rejects the null hypothesis when it is true.

A **type II error** occurs if one does not reject the null hypothesis when it is false.

The **level of significance** is the maximum probability of committing a type I error. This probability is symbolized by α (Greek letter **alpha**). That is, P(type I error) $= \alpha$.

H_0: $\mu = k$ $\quad \alpha = 0.10$, C.V. $= \pm 1.65$
H_1: $\mu \neq k$ $\quad \alpha = 0.05$, C.V. $= \pm 1.96$
$\quad \alpha = 0.01$, C.V. $= \pm 2.58$



Test value vs. critical value (C.V.)

Procedure Table

Solving Hypothesis-Testing Problems (Traditional Method)

STEP 1 State the hypotheses, and identify the claim.

STEP 2 Find the critical value(s) from the appropriate table in Appendix C.

STEP 3 Compute the test value.

STEP 4 Make the decision to reject or not reject the null hypothesis.

STEP 5 Summarize the results.

Example 9–5

The Medical Rehabilitation Education Foundation reports that the average cost of rehabilitation for stroke victims is \$24,672. To see if the average cost of rehabilitation is different at a particular hospital, a researcher selected a random sample of 35 stroke victims at the hospital and found that the average cost of their rehabilitation is \$25,226. The standard deviation of the population is \$3,251. At $\alpha = 0.01$, can it be concluded that the average cost of stroke rehabilitation at a particular hospital is different from \$24,672?

Source: Snapshot, *USA Today,* September 18, 1995.

Solution

STEP 1 State the hypotheses and identify the claim.

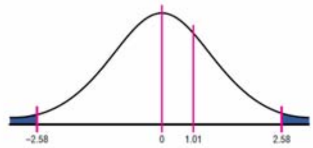
H_0: μ = \$24,672 and H_1: $\mu \neq$ \$24,672 (claim)

STEP 2 Find the critical values. Since $\alpha = 0.01$ and the test is a two-tailed test, the critical values are +2.58 and −2.58.

STEP 3 Compute the test value.

$$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{25{,}226 - 24{,}672}{3{,}251/\sqrt{35}} = 1.01$$

STEP 4 Make the decision. Do not reject the null hypothesis, since the test value falls in the noncritical region, as shown in Figure 9–15.



STEP 5 Summarize the results. There is not enough evidence to support the claim that the average cost of rehabilitation at the particular hospital is different from \$24,672.