1 Calculus I - Exercises

1. Using a limit of a difference quotient find an equation of the tangent line to the the graph of the function described by the equation $y = 3/x$ at the point (3,1). Graph both the function and the tangent line.

2. Let k be a constant and let f be the constant function $f(x) = k$ for all x. Prove using the definition of the derivative that $f'(x) = 0$ for all x.

3. Differentiate the following functions

   (a) $F(x) = -4x^{10}$

   (b) $g(t) = 5t^{\frac{3}{5}}$

   (c) $f(u) = \sqrt{u} - \dfrac{1}{\sqrt{u}}$

4. Differentiate the following functions using the product or quotient rules

   (a) $h(x) = (x^3 - 2x^2 + x + 1)(x^2 + 2x + 4)$

   (b) $G(x) = (x^4 + 2)^{87}$

   (c) $F(x) = (x^2 - 2)(5x^2 + 3)^3$

   (d) $h(x) = \dfrac{3x^3}{\sqrt{3 - x}}$

5. Differentiation using the chain rule.

   Recall that the chain rule is used in the cases that the function in question is a composition of two more simple functions. For instance F may be the function which squares a number adds 1 to the result and then takes the square root of everything. In this case F is the result of first an *inside* function $f(x) = x^2 + 1$ and an *outside* function, which given a number takes the square root. We might write $g(y) = \sqrt{y}$. Then $F(x) = g(f(x)) = g(x^2 + 1) = \sqrt{x^2 + 1}$. Remember, to differentiate such a function you need to:

   - first take the derivative of the outside function but leave whatever is in its bracket alone.
   - second multiply the result by the derivative of the inside function.

   We express the result as $F'(x) = g'(f(x)) \cdot f'(x)$ So, for the example where $F(x) = \sqrt{x^2 + 1}$, we calculate the derivative of the outside function as $g'(y) = \frac{1}{2y^{-\frac{1}{2}}} = \frac{1}{2\sqrt{y}}$.

Now make the substitution $y = x^2+1 = f(x)$. We get $g'(f(x)) = \frac{1}{2\sqrt{x^2+1}}$. To finish we need to multiply by $f'(x) = 2x$. The final result is: $F'(x) = g'(f(x)) \cdot f'(x) = \frac{2x}{2\sqrt{x^2+1}}$

Here are the problems. Differentiate each using the chain rule.

(a) $h(x) = \sqrt[3]{x^2 + 2x - 1}$

(b) $G(x) = (x^4 + 27)^{87}$

(c) $H(t) = (t^{\frac{1}{3}} + 1)^5$

(d) $g(x) = (\sqrt{x} + x^2)^{\frac{2}{5}}$

(e) $f(x) = \sqrt{\sqrt{x}}$

6. Differentiate the following using the product or quotient rules and the chain rule.

- $F(x) = (x^3 - 2)(5x^2 + 3)^{33}$
- $f(x) = \dfrac{3x^2}{\sqrt{3 - x}}$
- $g(t) = \dfrac{(t^2 + 2)^8}{t^{\frac{3}{7}} + 2}$
- $h(z) = (z^3 + z^2 + 1)^{26}(\sqrt{z^2 + 1} + z)^7$

7. Find an equation of the tangent to the curve described by the equation $y = \sqrt[3]{x^2 - 2x + 1}$ at the point $(2, 1)$ on the curve. Use a computer graphing program to graph the equation and the tangent line.