Exercise 1 (Fibonacci Stuff)

(a) Write down the first 6 terms of the Fibonacci sequence, and if f_n is the n^{th} term of the Fibonacci sequence, calculate $\frac{f_n}{f_{n-1}}$ for $n = 5, 6, 7$

(b) Solve the equation $x^2 - x - 1 = 0$ using the quadratic formula. Present your answers both precisely in terms of square roots and as decimals.

(c) Given a rectangle whose smaller side has length = 1 and whose larger side has length = x. Adjoin to the larger side a square whose sides are of length x, obtaining a larger rectangle. Suppose the rectangle is such that length divided by width is the same for both the smaller and the larger rectangles. What is x? *Rectangles with this unique shape are called the "golden rectangles" and have played an important part in art and architecture throughout the ages.* See figure 1.

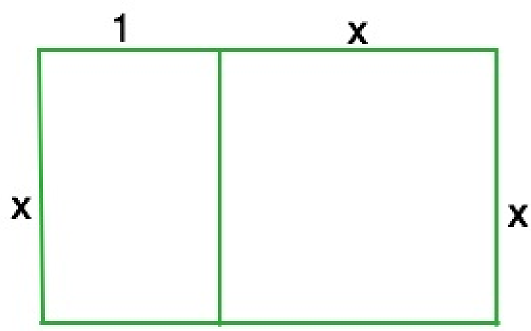

Figure 1: A rectangle of dimensions width = 1 and length = x with adjoined square with sides of length x

(d) Given a line segment with end points A and B let C be a point on the line between A and B which divides the line segment into a larger and a smaller part. Suppose the larger part, say AC has length equal to x and the smaller part, CB, has length equal to one. Further suppose that the ratio of the length of the complete segment to that of the larger part equals the ratio of the length of the larger part to that of the smaller - that is AB/AC = AC/CB. What is x?

(e) Take your shoes off and measure your height. Next measure the distance from the floor to your belly button. Divide the first number by the second.

Exercise 2

(a) Find the first 5 terms of the sequence a_n where $a_n = \frac{1}{1+a_{n-1}}$ for $a_1 = 1$.

(b) Find the 4^{th} partial sum S_4 for the sequence defined by $a_n = \frac{2}{3^n}$.

(c) Find the sums $\Sigma_{k=2}^{4} \frac{k}{3}$ and $\Sigma_{n=1}^{6} \frac{2}{n}$

Exercise 3

(a) Find the common difference, the fifth term, the n^{th} term, and the 100th term of the arithmetic sequence

$$11, 8, 5, 2, \cdots$$

(b) Find the partial sum S_n of the arithmetic sequence that satisfies: $a = 3,\ d = 2,\ n = 12$

(c) A woman gets a job with a salary of 50,000 a year. She is promised a 2800 raise each year. Find her total earnings at the end of 15 years - use the formula for the sum of an arithmetic sequence.

Exercise 4

(a) Find the common ratio, the fifth term, and the n^{th} term of the two geometric sequences

1. $1, \sqrt{2}, 2, 2\sqrt{2}, \cdots$
2. $-8, -2, -\frac{1}{2}, -\frac{1}{8}, \cdots$

(b) Find the the sum S_n of the geometric sequence that satisfies $a = \frac{2}{3}, r = \frac{1}{3}, n = 4$

Exercise 5

Find the sum of the following infinite geometric series

1. $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \cdots$
2. $3 - \frac{3}{2} + \frac{3}{4} - \frac{3}{8} \cdots$

Exercise 6

A ball is dropped from a height of 100 feet. The elasticity of the ball is such that it rebounds three-quarters of the distance it has fallen.

- How high does the ball rebound on the 6^{th} bounce?
- Find a formula for how high the ball rebounds onthe n^{th} bounce.
- How far does the ball travel up and down before it comes to a rest?

Hint Draw a picture of the ball bouncing - bearing in mind that after hitting the ground, it bounces up a certain amount and then falls down the same amount. Deduce a geometric series that starts with the first bounce up - bearing in mind that by doing so you have ignored the first fall of 100 feet.

Exercise 7

Suppose a sequence a_n converges to a limit L. Let k be any real number. Prove using the definition of the limit of a sequence that the sequence ka_n converges to kL.

Exercise 8

Suppose that the series $\Sigma_{n=1}^{\infty} a_n$ sums to a number S. Let k be any real number. Prove, using the definition of what it means for a series to converge i.e. the sum makes sense, that the series $\Sigma_{n=1}^{\infty} ka_n$ sums to kS.