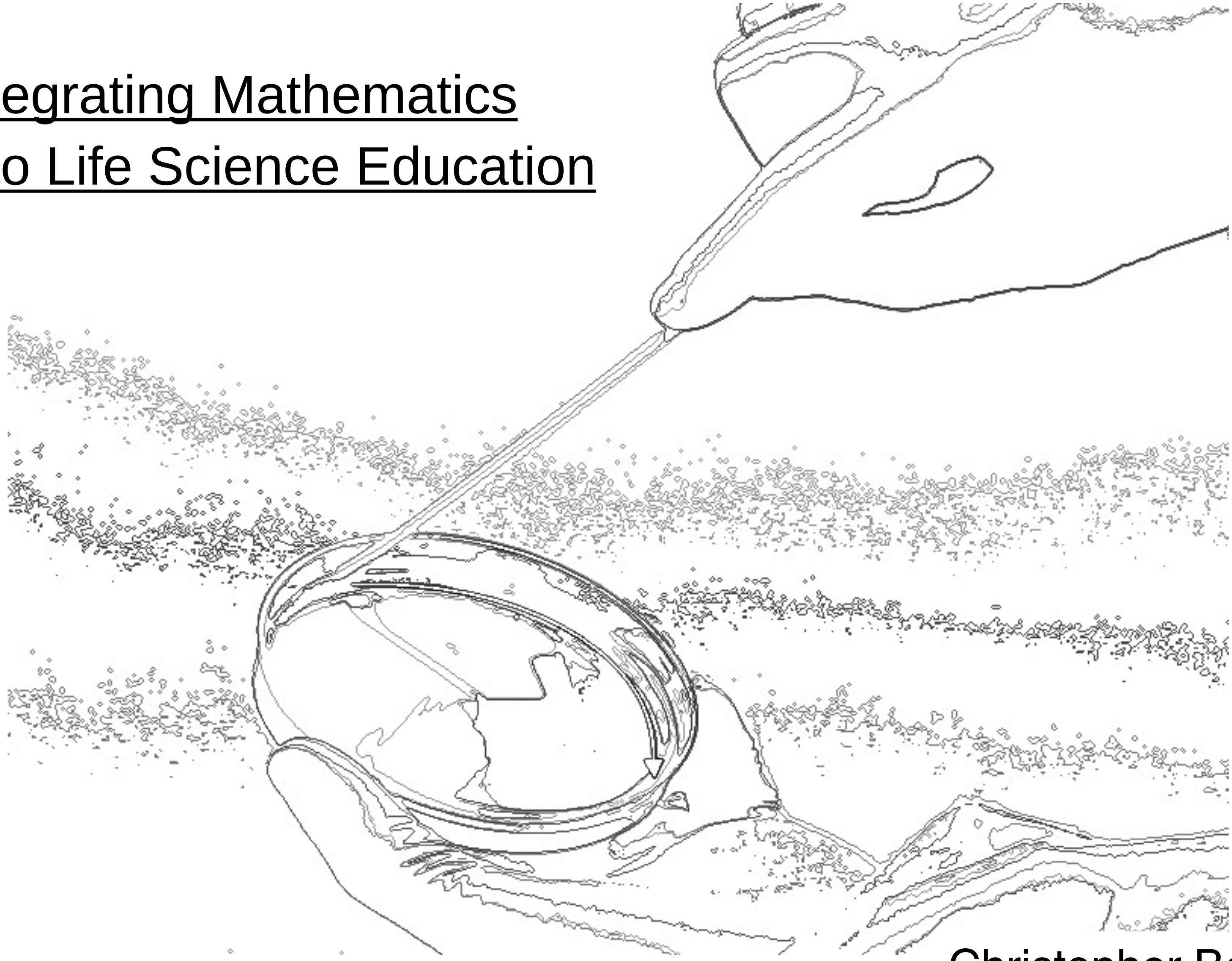
Integrating Mathematics Into Life Science Education



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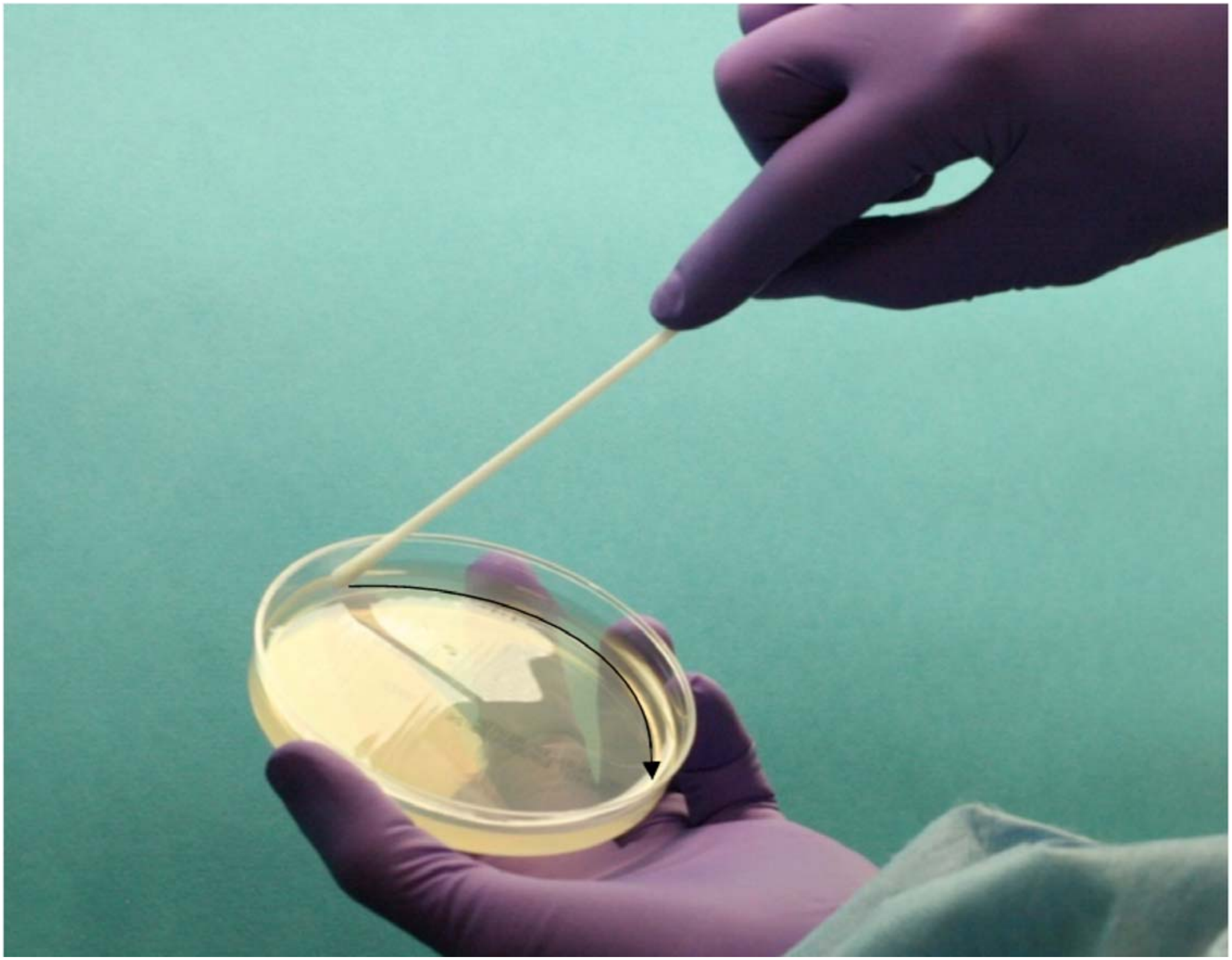
ASMCUE (05/10)



ASM MicrobeLibrary.org (Jan Hudzicki, University of Kansas Medical Center)

University of Arizona

Bio-Math Group

MISSION: The primary purpose of the Bio-Math Committee is to advise the Mathematics Department at the University of Arizona in the design of novel biology-oriented content for three of its courses that are recommended for students who plan majors in one of the College of Science Biology Departments.

- Calculus/Differential Equations (Math 250 A&B)
- *{Calclus-based}* Statistics (Math 363)
- Analytical Approaches to ‘Bio’ Fundamentals (PSIO 472/572)

Financial Support: National Science Foundation, Howard Hughes Medical Institute

Educational Dilemma?

“Dramatic advances in biological understanding, coupled with equally dramatic advances in experimental techniques and computational analyses, are transforming the science of biology. ... Even though most biology students take several years of prerequisite courses in mathematics and the physical sciences, these students have too little education and experience in quantitative thinking and computation to prepare them to participate in the new world of quantitative biology. At the same time, advanced physical science students who become interested in biological phenomena can find it surprisingly difficult to master the complex and apparently unconnected information that is the working knowledge of every biologist.”

Bialek & Botstein (2004)

Point of Focus: Mathematics & Biology

"Dramatic advances in biological understanding, coupled with equally dramatic advances in experimental techniques and computational analyses, are transforming the science of biology. ... Even though most biology students take several years of prerequisite courses in mathematics and the physical sciences, these students have too little education and experience in quantitative thinking and computation to prepare them to participate in the new world of quantitative biology. At the same time, advanced physical science students who become interested in biological phenomena can find it surprisingly difficult to master the complex and apparently unconnected information that is the working knowledge of every biologist."

Bialek & Botstein (2004)

$\Rightarrow$

We need to change the way we educate 21st century life scientists

Question(s) In a Nutshell:

A. Given rapid advancements in technology & knowledge-base, how is student learning/thinking evolving?

B. Do these changes differ across subjects (e.g., *mathematics* vs. *biology* vs. *geography*)?

C. As such, how do educators need to change their pedagogy? Different for K-12 vs. University level?

Outline:

Attitudes Questionnaire

Overview of need for BioMath development

Case Study

(diffusion, Brownian motion, anti-microbials)

Attitudes Questionnaire

Possible Tactic(s)?

Given ubiquity of tools at hand, shift in emphasis upon *critical thinking* and *attitude*

⇒ What does critical thinking mean?

1. Retention of concepts/ideas & integration into subsequent courses

2. Improvement of analytic and scientific reasoning skills

3. Shift in student's attitudes so to demand a deeper conceptual understanding of issues encountered both inside and outside the classroom, regardless of the subject

Idea: Think of mathematics as a means to develop critical thinking skills via *interdisciplinary* connections

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Perspectives

Towards Improving the Integration of Undergraduate Biology and Mathematics Education

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Arguments have recently asserted the need for change in undergraduate biology education, particularly with regard to the role of mathematics. The crux of these protests is that rapidly developing technology is expanding the types of measurements and subsequent data available to biologists. Thus future generations of biologists will require a set of quantitative and analytic skills that will allow them to handle these types of data in order to tackle relevant questions of interest. In this spirit, we describe here strategies (or lessons learned) for undergraduate educators with regard to better preparing undergraduate biology majors for the new types of challenges that lay ahead. The topics covered here span a broad range, from classroom approaches to the administrative level (e.g., fostering inter-departmental communication, student advising) and beyond. A key theme here is the need for an attitude shift with regard to mathematics education by both students and faculty alike. Such a shift will facilitate the development and implementation of new teaching strategies with regard to improving integration of mathematics and biology pedagogy.

KEY WORDS: Biomath, mathematics education, bio2010

MOTIVATION

Recent reports have called for change in how undergraduate mathematics education is approached for students in biology (6). A compelling argument was made by Bialek and Botstein that the traditional path towards fostering quantitative biologists – having students from the physical/mathematical/engineering sciences get involved in biological problems at the graduate level (or beyond) – was no longer sufficient (5). They argued that biological sciences were getting too inherently complex to effectively learn the biology and the interconnections across various fields of study at a late stage; a more integrated approach was required early on at the undergraduate level. Another recent publication lends support (2), indicating that analytical content knowledge alone for those finishing secondary education does not necessarily correlate well to a student's scientific reasoning ability.

Clearly, there is a compelling need for future generations of biologists with strong quantitative and analytic reasoning skills (6). For example, mathematical-based models can serve to tie together the vast swaths of biological data that are increasingly coming to light. Given the complexity of biological systems, such models serve to provide a coherent and interpretable framework with which to tie together empirical observations. One example is the field of neuroscience, where models can be critical for determining future research directions (1).

The goal of this Perspectives piece is to outline a set of strategies and priorities for educators who are interested in the integration of quantitative and analytical reasoning skills into biology education by means of developing new biology–mathematics curricula (BioMath). A critical aspect for developing and implementing new strategies is facilitating an attitude shift in both students and faculty alike with regard to the perception and learning of mathematics. While the target audience is undergraduate educators, presumably much of the present discussion applies to high school and even graduate education. For details with regard to developing specific course content, several texts provide an excellent starting point at the undergraduate level (and beyond) (3, 22, 12, 9, 17, 14, 8).

The ideas described here derive from experiences with the BioMath committee at the University of Arizona. This committee, comprised of faculty from several departments as well as administrators and students, developed and implemented a new three-semester mathematics sequence offered to incoming life sciences freshmen. The courses, covering integral calculus, differential equations and (calculus-based) statistics, were designed with several goals in mind:

1. Retention of mathematical concepts and integration into subsequent science courses
2. Improvement of analytic and scientific reasoning skills
3. Shift in student's attitudes towards demanding a deeper conceptual understanding of issues encountered both inside and outside the classroom, regardless of the subject

A fourth course for upperclassmen was also developed,

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Short Refresher: *Types* of Mathematics

Differential equations

Discrete Mathematics

Calculus

Linear Algebra

Statistics & Probability

Stochastic Dynamics

Vector Calculus

Fourier Analysis

Fact: If you don't use it, you lose it

Idea: Think of mathematics as a means to develop critical thinking skills via *interdisciplinary* connections

Refreshers? Inter-departmental connections?

→Folks only have so much time…..

Short Excursion: Pre-Class Primer

⇒ Lateral Thinking Problems

- Encourages creativity and that it is okay (even good!) to *think outside the box*

- Commonly regarded as simple in hindsight, but can be incredibly difficult to tackle (like the real world!)

It's not an easy life being a zoo keeper. I can't begin to imagine how they tell all those penguins apart.

To see if you should apply to your local zoo for a job, I'll supply an initial aptitude test. Here is a pair of pictures of a panda, and there are two clearly visible differences between them.

Identify both of them.

Source: 'Mind Benders: Adventures in Lateral Thinking' by David J. Bodycombe

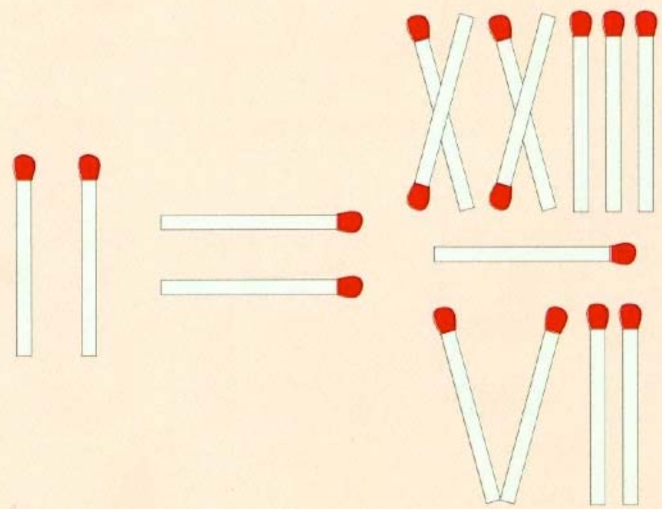
Two people were discussing the matchstick puzzle shown below. The idea was to move only one match to make the statement correct.

"I reckon it's impossible to make it exactly correct," claimed Kevin, "unless you cheat by turning the equals sign into a 'not equal to' sign."

"Maybe," said Mandy, "but I can make it approximately correct." She made her move and the result was only a few percent from being exactly correct.

"Aha!" exclaimed Kevin. "I can move the same match as you've just moved to make the statement even closer to being exactly correct."

Can you work out what the two solutions were?

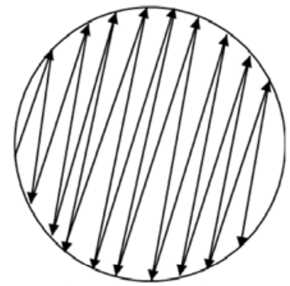


Source: 'Mind Benders: Adventures in Lateral Thinking'
by David J. Bodycombe

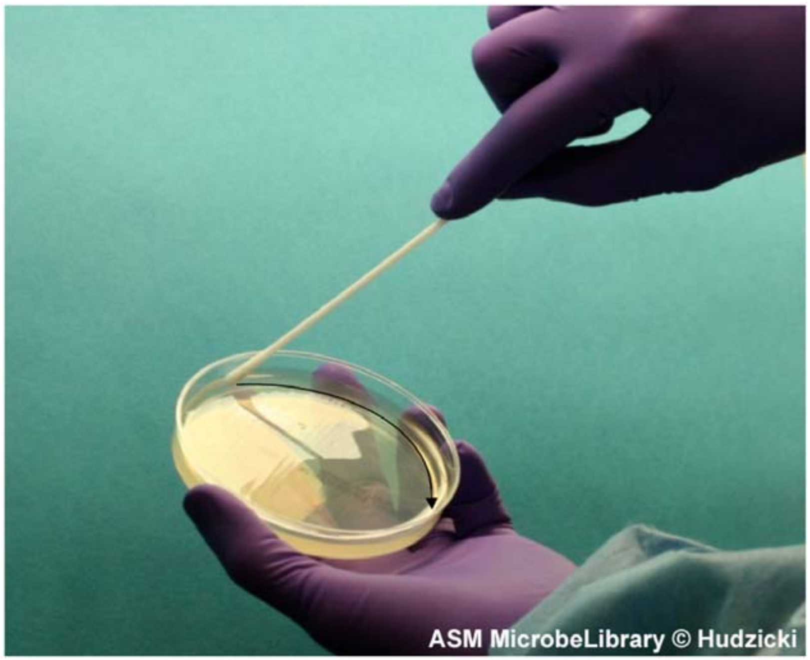
Case Study: Diffusion

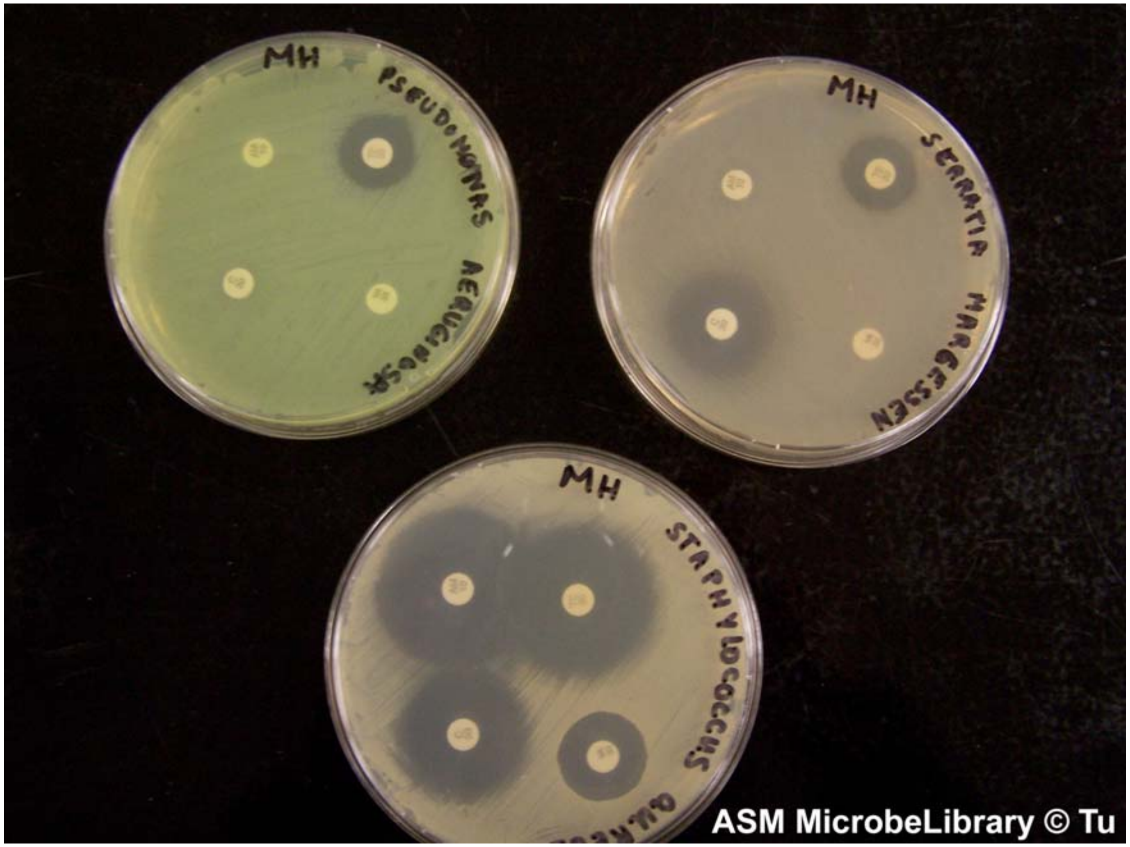
e.g., Kirby-Bauer Disk Diffusion Susceptibility Test

(description & pictures via ASM *MicrobeLibrary.org*;
Jan Hudzicki, University of Kansas Medical Center)



ASM MicrobeLibrary © Hudzicki





"Results can be read after 18 hours of incubation unless you are testing Staphylococcus against oxacillin or vancomycin, or Enterococcus against vancomycin. Read the results for the other antimicrobial disks then reincubate the plate for a total of 24 hours before reporting vancomycin or oxacillin."

Questions:

- Why does it takes so long?

- Can we understand and quantify the time course of the disk's radius?

- What if the temperature was changed?

Mathematical Topics To Touch Upon

Differential equations

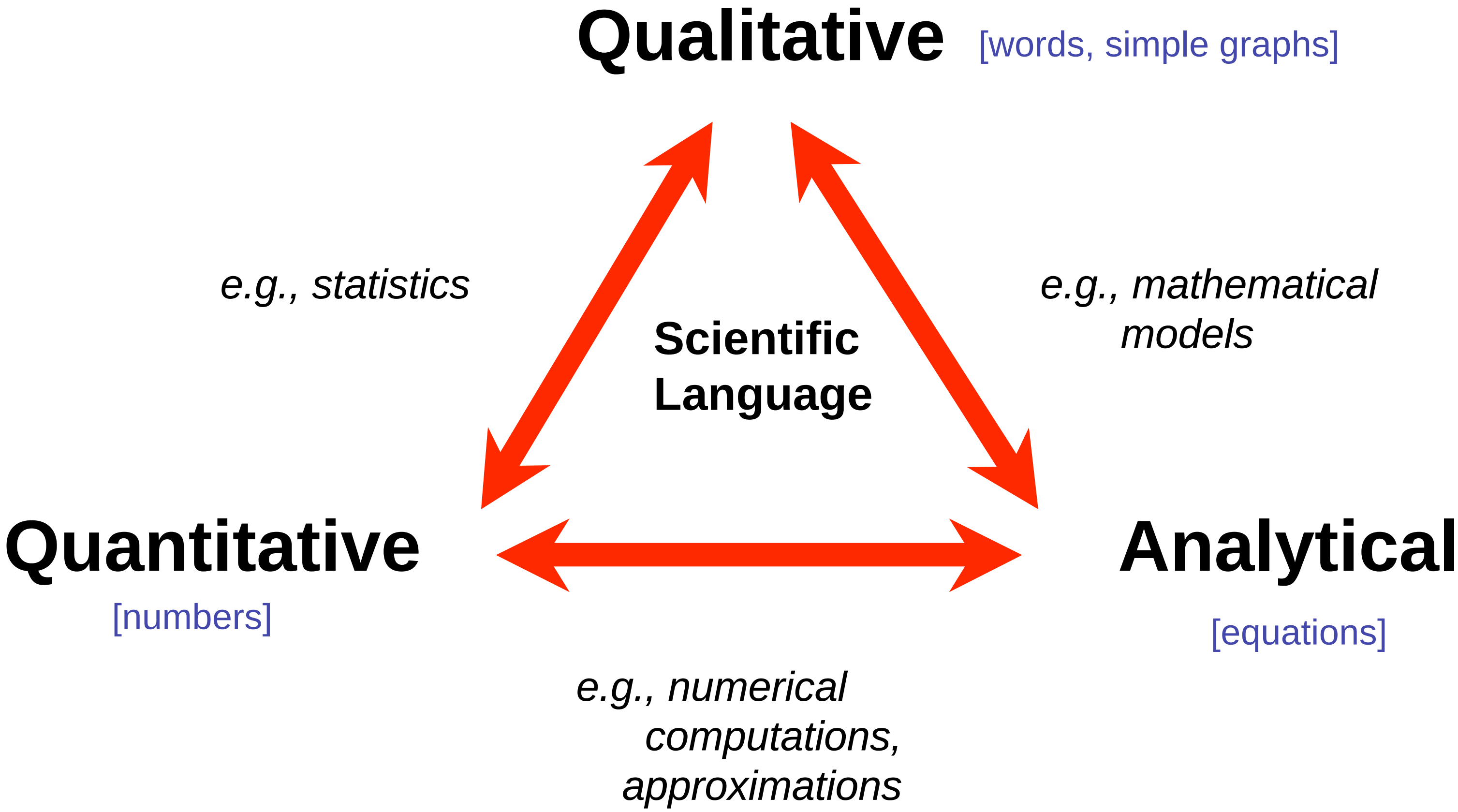
Calculus
(e.g., limits, derivatives)

Statistics, Probability, &
Gaussian Distributions (e^{-x^2})

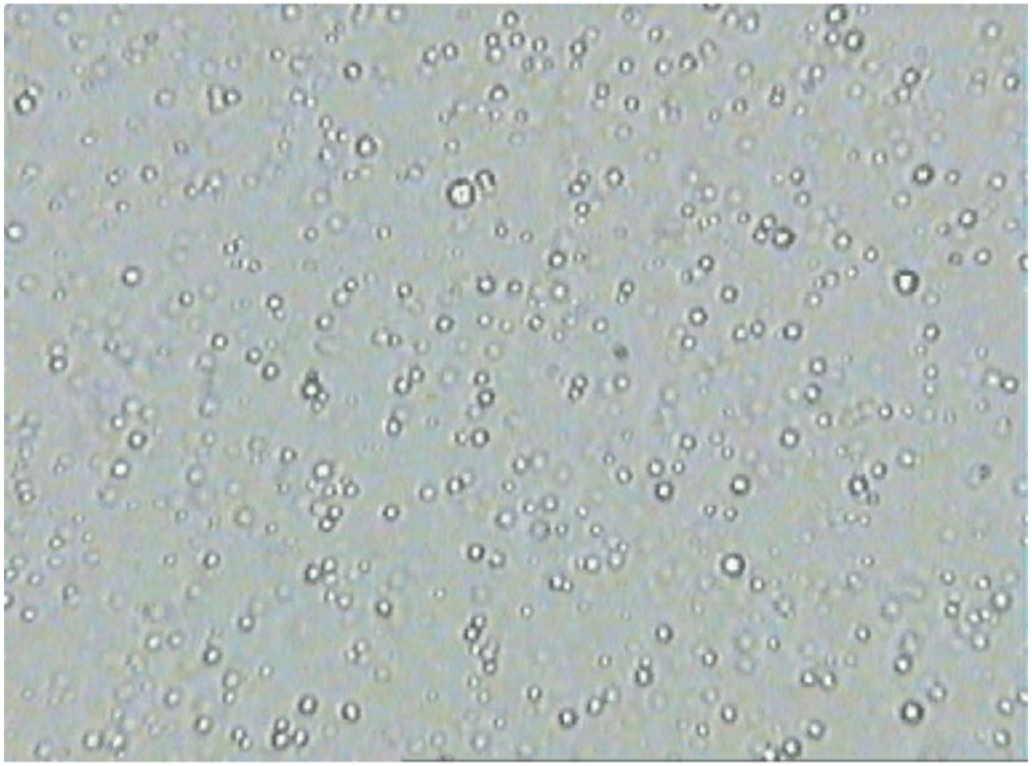
Stochastic Dynamics

Multi-Variable Functions

Set of Definitions:



Short Excursion: Microscopic Basis for Diffusion



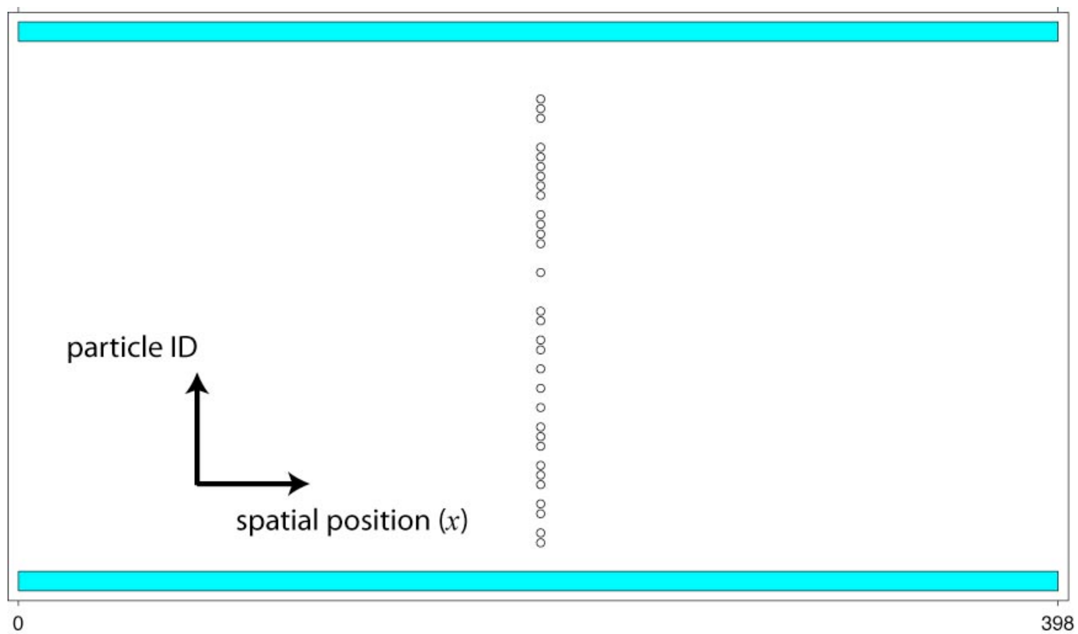
⇒ Brownian motion

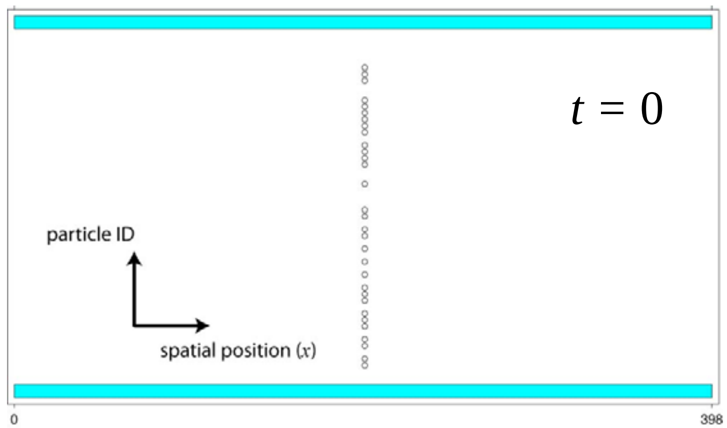
http://www.microscopy-uk.org.uk/dww/home/hombrown.htm

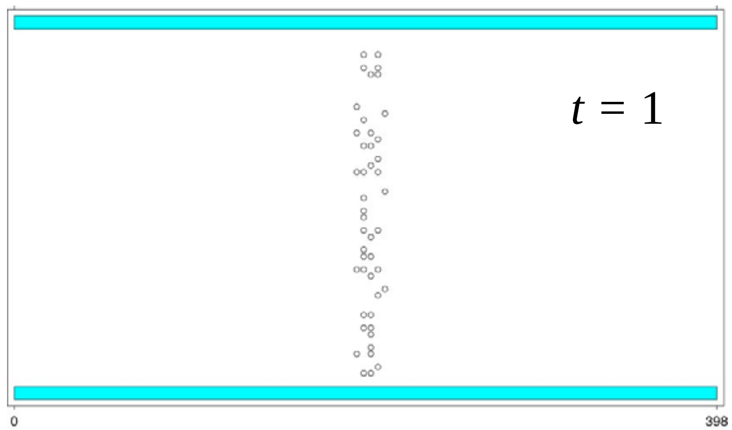
Short Excursion: Brownian Motion

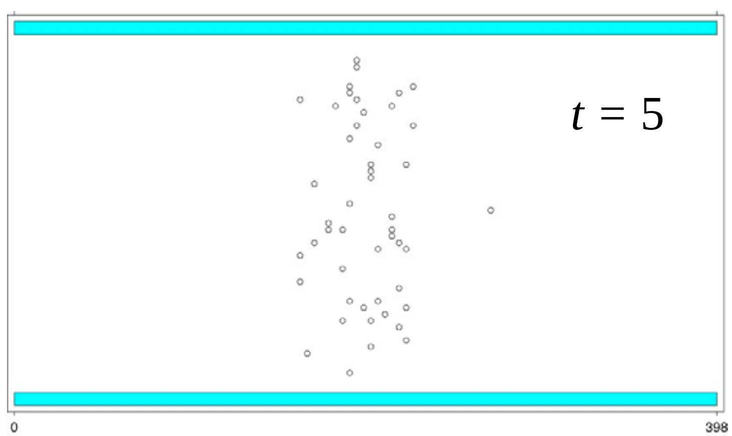
Brownian motion $\Rightarrow$ ‘Random Walker’ (1-D)

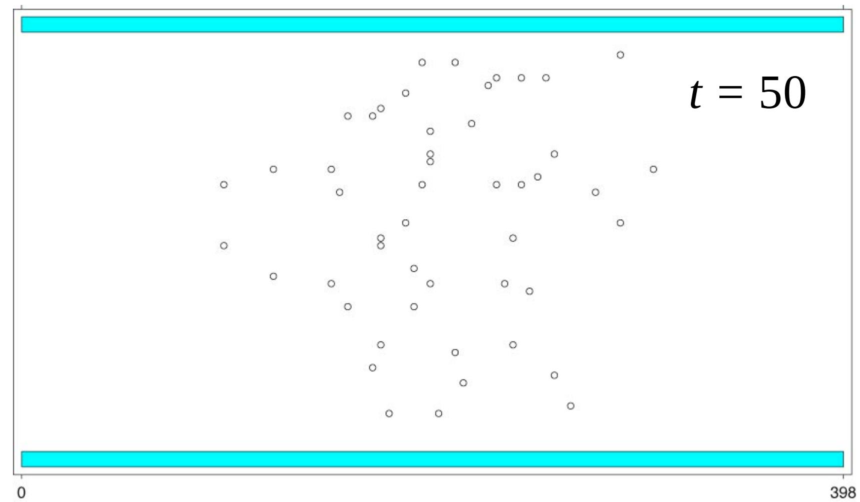
Ensemble of Random Walkers

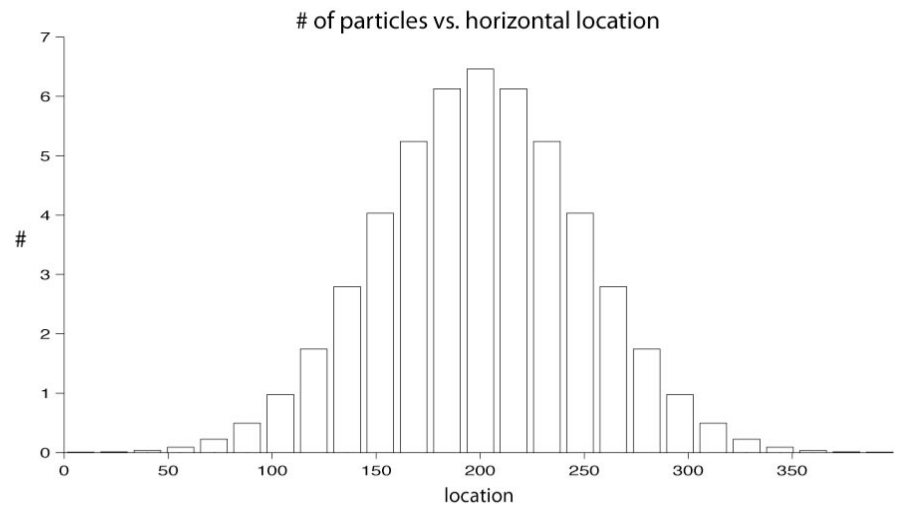



t = 0
particle ID
spatial position (x)
0
398


t = 1
0
398


t = 5
0
398


t = 50
0
398


# of particles vs. horizontal location
#
location
0
1
2
3
4
5
6
7
0
50
100
150
200
250
300
350

5. ***Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen; von A. Einstein.***

In dieser Arbeit soll gezeigt werden, daß nach der molekularkinetischen Theorie der Wärme in Flüssigkeiten suspendierte Körper von mikroskopisch sichtbarer Größe infolge der Molekularbewegung der Wärme Bewegungen von solcher Größe ausführen müssen, daß diese Bewegungen leicht mit dem Mikroskop nachgewiesen werden können. Es ist möglich, daß die hier zu behandelnden Bewegungen mit der sogenannten „Brownschen Molekularbewegung" identisch sind; die mir erreichbaren Angaben über letztere sind jedoch so ungenau, daß ich mir hierüber kein Urteil bilden konnte.

Wenn sich die hier zu behandelnde Bewegung samt den für sie zu erwartenden Gesetzmäßigkeiten wirklich beobachten läßt, so ist die klassische Thermodynamik schon für mikroskopisch unterscheidbare Räume nicht mehr als genau gültig anzusehen und es ist dann eine exakte Bestimmung der wahren Atomgröße möglich. Erwiese sich umgekehrt die Voraussage dieser Bewegung als unzutreffend, so wäre damit ein schwerwiegendes Argument gegen die molekularkinetische Auffassung der Wärme gegeben.

§ 1. Über den suspendierten Teilchen zuzuschreibenden osmotischen Druck.

Im Teilvolumen V^* einer Flüssigkeit vom Gesamtvolumen V seien z-Gramm-Moleküle eines Nichtelektrolyten gelöst. Ist das Volumen V^* durch eine für das Lösungsmittel, nicht aber für die gelöste Substanz durchlässige Wand vom reinen Lösungs-

Annus Mirabilis papers

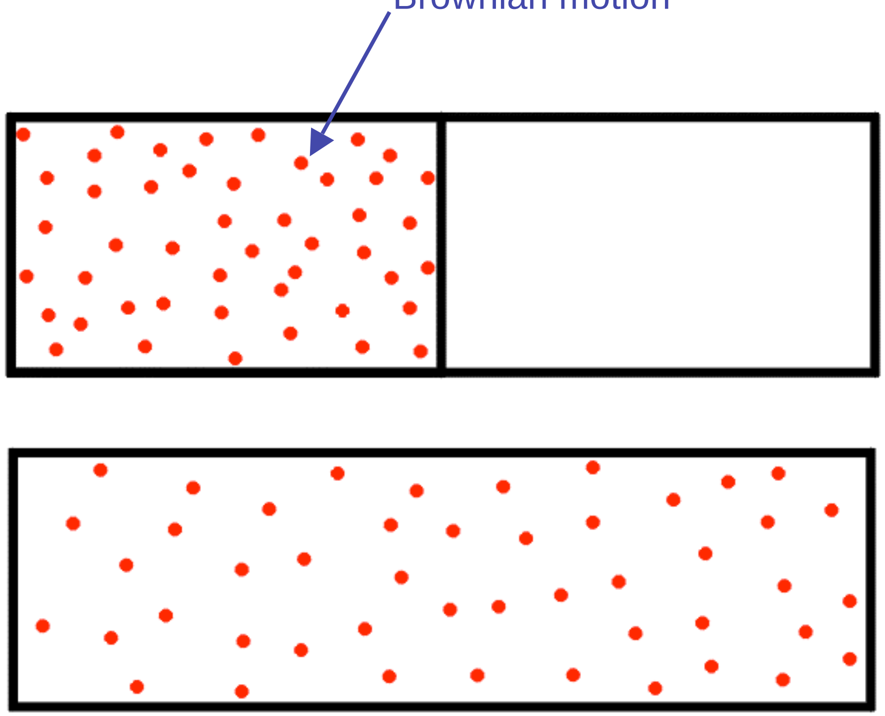
"On the Motion of Small Particles Suspended in a Stationary Liquid, as Required by the Molecular Kinetic Theory of Heat"

A. Einstein (1905)

Annalen der Physik

http://en.wikipedia.org/wiki/File:Einstein_patentoffice.jpg

Diffusion



Ex.1 Diffusion is a key process in how antimicrobial agent fans out

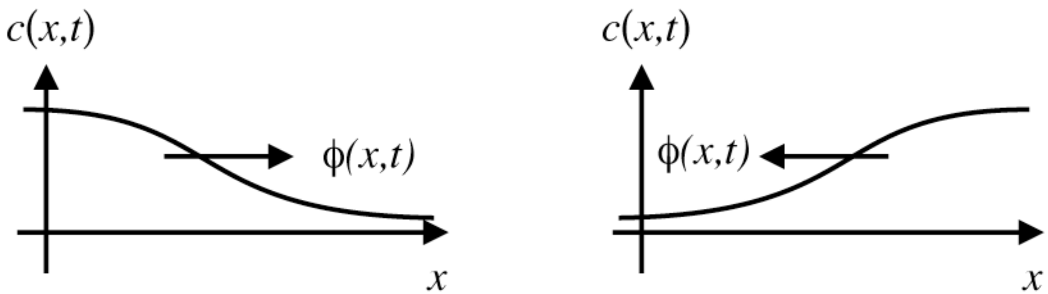
Ex.2 Diffusion is a key process in how neurons transmit information (i.e., synapses)

http://en.wikipedia.org/wiki/Image:Entropie.png

Diffusion (1-D)

- Thomas Graham (Scottish chemist, ~1828-1833)

[pioneered the concept of dialysis]



- Adolf Fick (German physiologist, ~1855)

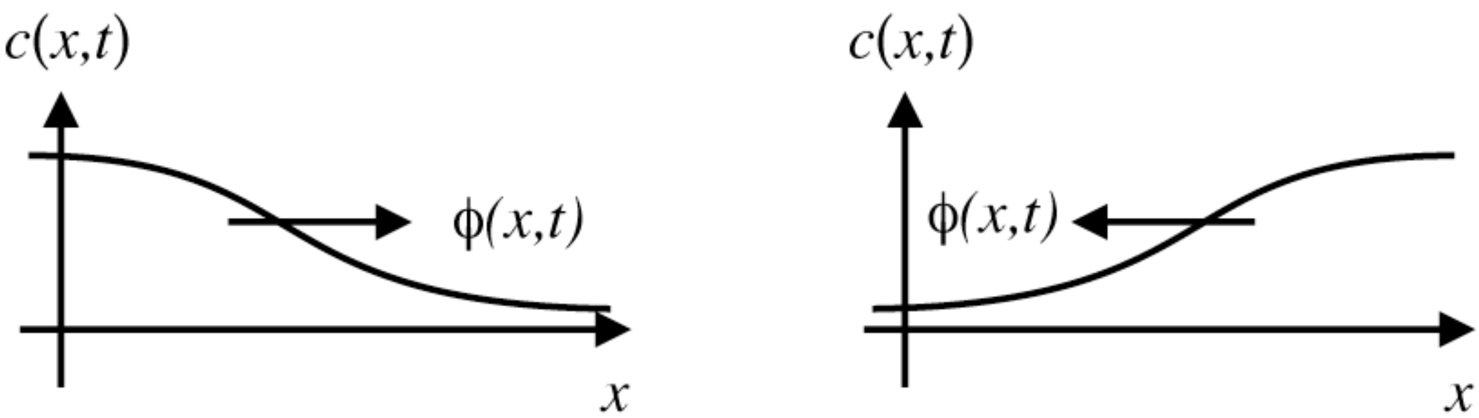
[his nephew (same name) actually was the first to successfully put a contact lens on a person in 1888!]

“ A few years ago, Graham published an extensive investigation on the diffusion of salts in water, in which he more especially compared the diffusibility of different salts. It appears to me a matter of regret, however, that in such an exceedingly valuable and extensive investigation, the development of a fundamental law, for the operation of diffusion in a single element of space, was neglected, and I have therefore endeavoured to supply this omission.”

- A. Fick (1855)

Diffusion (1-D)

From Graham's observations (~1830):



$$c(x,t)$$

Concentration - of solute in solution [*mol/m³*]

$$\phi(x,t)$$

Note: flux is a vector!

Flux - net # of moles crossing per unit time t through a unit area perpendicular to the x-axis [$mol/m^2 \cdot s$]

$$x,t$$

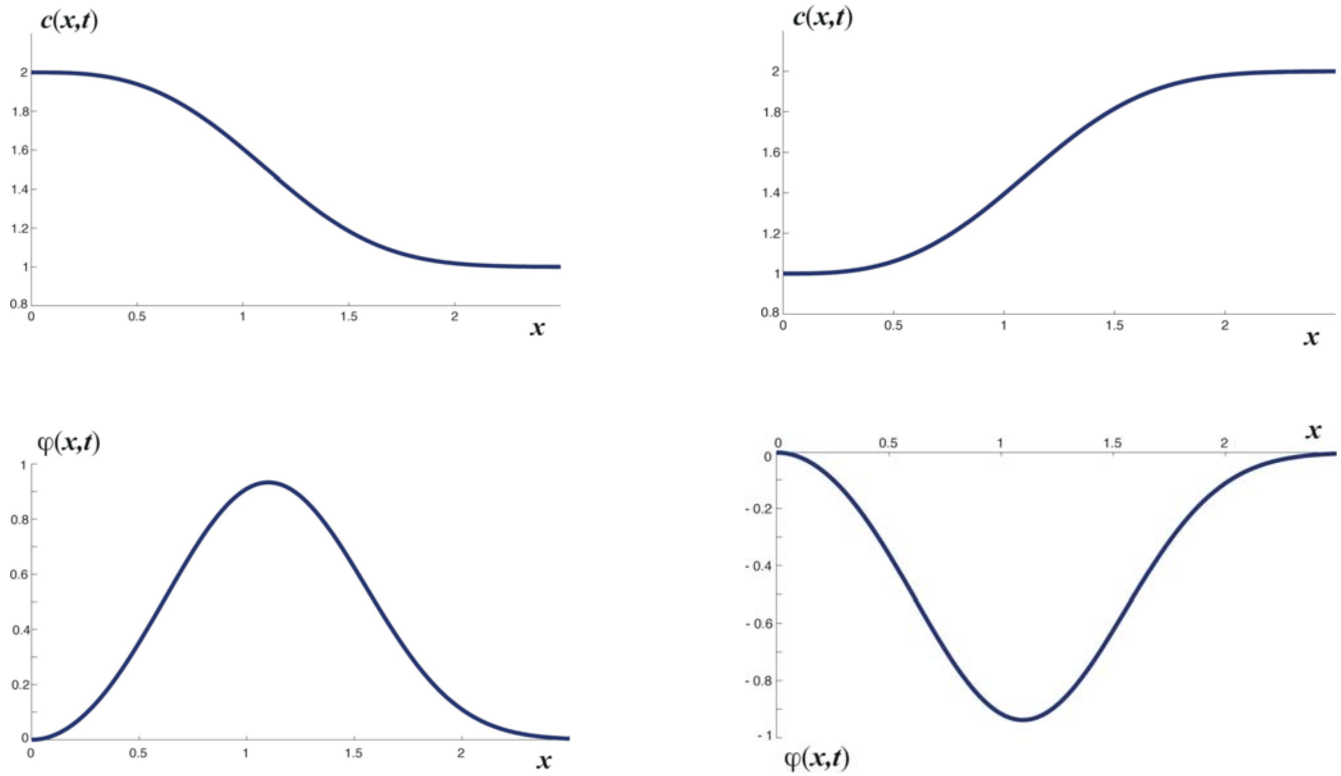
Position [*m*], Time [*s*]

Fick's First Law (1-D)

Profile 1 | Profile 2

Concentration (c)

Flux (φ)



Position (x)

$$\phi(x,t) \propto -\frac{\partial c(x,t)}{\partial x}$$

Diffusion Constant D

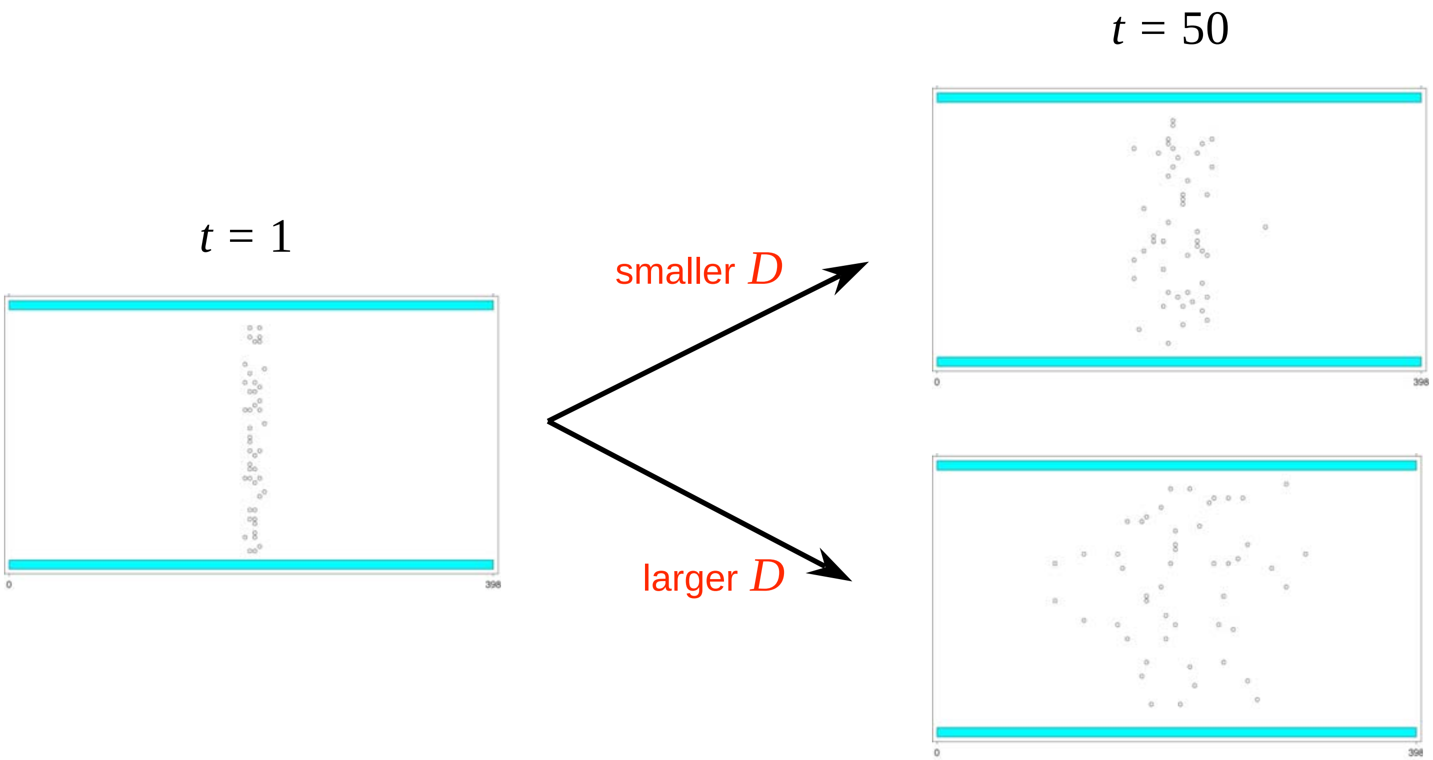
$$\phi(x,t) \propto -\frac{\partial c(x,t)}{\partial x}$$

constant of proportionality?

$$\phi(x,t) = -D\frac{\partial c(x,t)}{\partial x}$$

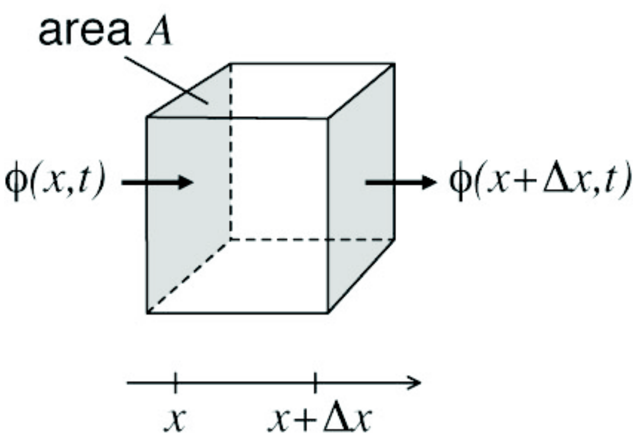
- diffusion constant is always positive (i.e., $D > 0$)
- determines time it takes solute to diffuse a given distance in a medium
- depends upon both solute and medium (solution)
- *Stokes-Einstein relation* predicts that D is inversely proportional to solute molecular radius

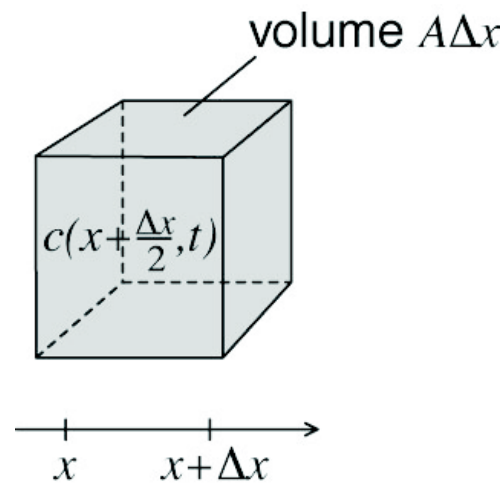
Diffusion Constant D



Continuity Equation

⇒ imagine a cube (with face area A and length Δx) and a time interval Δt



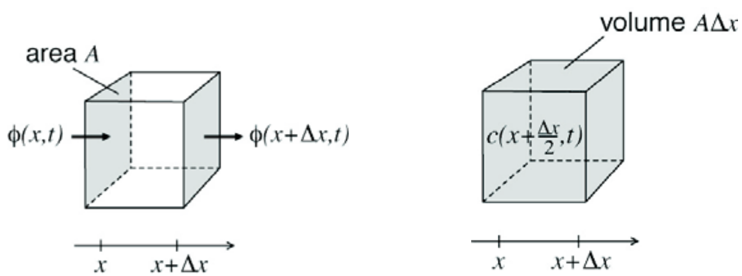


solute entering from ***left*** - solute exiting from ***right*** = change in amount of solute ***inside*** cube

(during time interval $[t, t+\Delta t]$) (during time interval $[t, t+\Delta t]$)

$$A\,\Delta t\,\phi(x,t) \qquad\qquad A\,\Delta x\,c(x,t)$$

$$\lim_{\Delta t, \Delta x \to 0} \quad \Longrightarrow \quad \boxed{\frac{\partial \phi}{\partial x} = -\frac{\partial c}{\partial t}}$$



⇒ conservation of mass within the context of our imaginary cube yielded the *continuity equation*

Diffusion Equation

1. Fick's First Law:

$$\phi = -D\frac{\partial c}{\partial x}$$

+

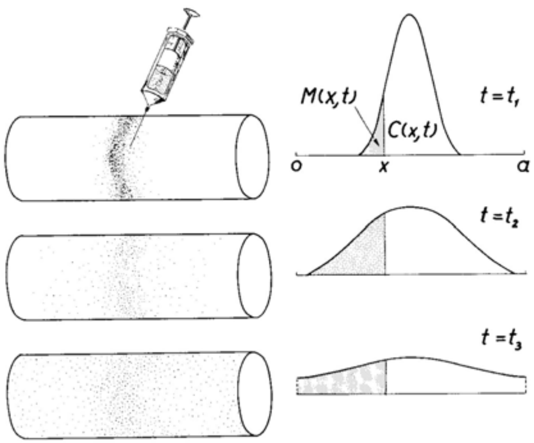
2. Continuity Equation:

$$\frac{\partial \phi}{\partial x} = -\frac{\partial c}{\partial t}$$

$$\boxed{\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}}$$

Partial differential equation (PDE);
can be hard to solve!!

Diffusion Processes

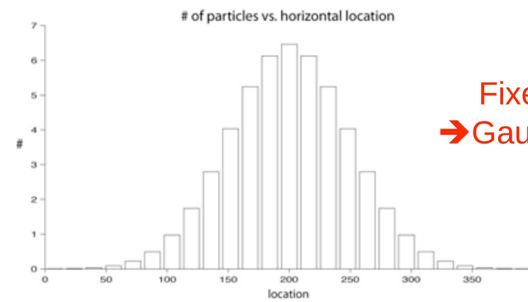


Batschelet Fig.12.5

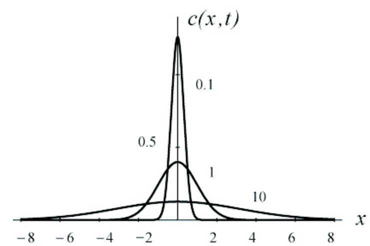
Point Source!

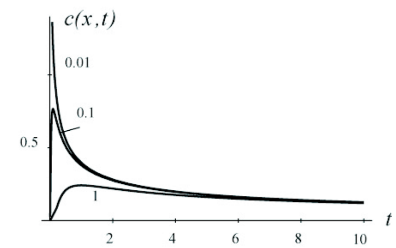
Solution to Diffusion Eqn. for point source (for $t > 0$)

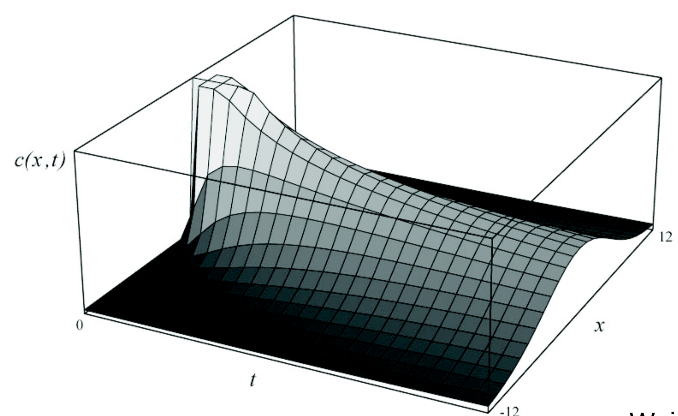
$$c(x,t) = \frac{n_o}{\sqrt{4\pi D t}} e^{-x^2/4Dt}$$



Fixed t
➔Gaussian!







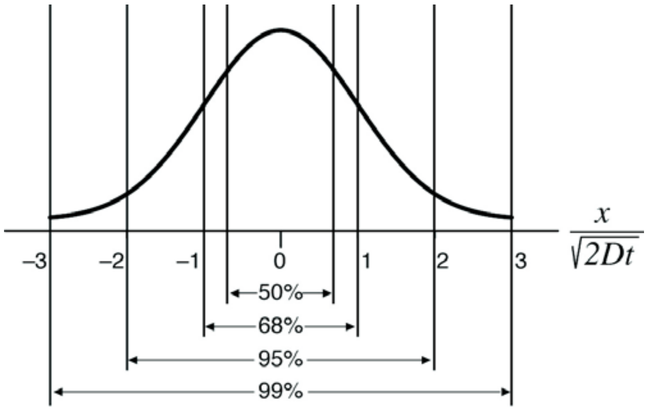
Weiss Fig.3.14 (modified)

Importance of Scale

$$c(x,t) = \frac{n_o}{\sqrt{4\pi D t}} e^{-x^2/4Dt}$$

Gaussian function with zero mean and standard deviation:

$$\sigma = \sqrt{2Dt}$$



Question: How long does it take ($t_{1/2}$) for ~1/2 the solute to move at least the distance $x_{1/2}$?

$$\frac{x_{1/2}}{\sqrt{2Dt_{1/2}}} \approx \frac{2}{3} \quad \Longrightarrow \quad \boxed{t_{1/2} \approx \frac{x_{1/2}^2}{D}}$$

For small solutes
(e.g. K^+ at body temperature)

$$D \approx 10^{-5} \ \frac{\text{cm}^2}{\text{s}}$$

	$x_{1/2}$	$t_{1/2}$
membrane sized	10 nm	$\frac{1}{10}$ μsec
cell sized	10 μm	$\frac{1}{10}$ sec
dime sized	10 mm	10^5 sec ≈ 1 day

Freeman

Importance of Scale

Implication #1

Question: Stepping barefoot on a tack, if neurons carried information to the solely via diffusion, how long would it take to realize the problem?

Longest neuron in body (foot to base of spinal cord): ~0.8 m

$$t_{1/2} \approx \frac{x_{1/2}^2}{D}$$

For small solutes
(e.g. K^+ at body temperature)

$$D \approx 10^{-5} \ \frac{\mathrm{cm}^2}{\mathrm{s}}$$

⇒ Roughly 20 years!!

Implication #1: Neurons needed to find ways to communicate faster axonally (diffusion okay across synapses!)

Importance of Scale

Implication #2

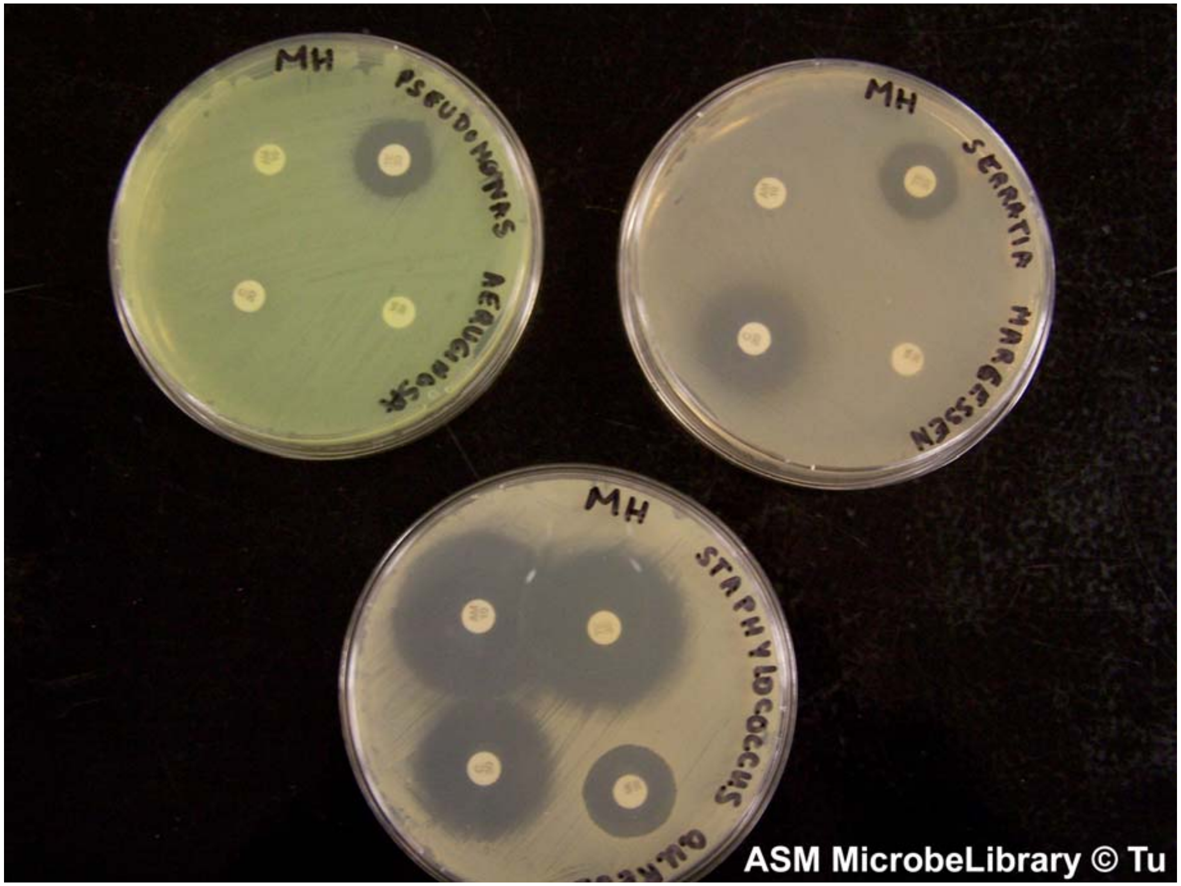
Question: If you are a bacteria, how long is it going to take to get to where you want to go via diffusion?

Harder question to answer, but note two key desires:

1. Want to be able to choose direction you head in

2. Want to get there relatively quickly if possible

Implication #2: Bacteria learned how to swim!



Questions:

Answers

- Why does it takes so long?

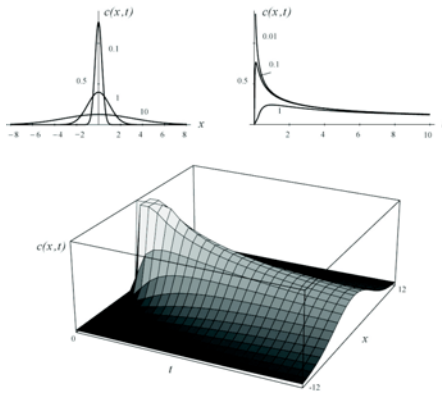
Diffusion slow on macroscopic scales

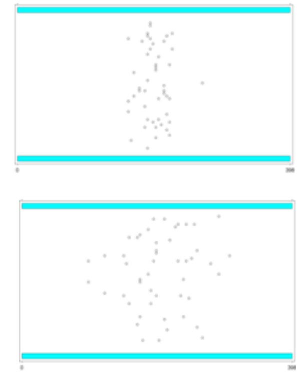
- Can we understand and quantify the time course of the disk's radius?

Yes! (though obviously things are a bit more complicated)

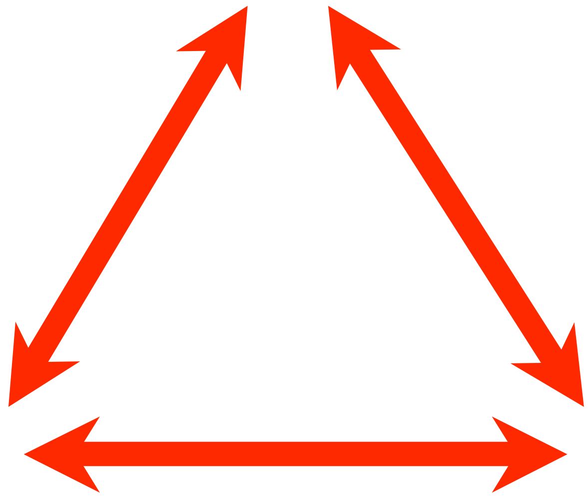
- What if the temperature was changed?

Changes diffusion constant (among other things)





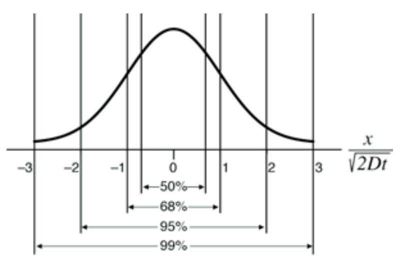
Qualitative



Quantitative

Analytical

	$x_{1/2}$	$t_{1/2}$
membrane sized	10 nm	$\frac{1}{10}$ μsec
cell sized	10 μm	$\frac{1}{10}$ sec
dime sized	10 mm	10^5 sec ≈ 1 day



$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}$$

$$t_{1/2} \approx \frac{x_{1/2}^2}{D}$$

$$c(x,t) = \frac{n_o}{\sqrt{4\pi Dt}}e^{-x^2/4Dt}$$

Your turn:

A. Form small groups (~3 people)

B. Determine an important concept that would be covered over several lectures

C. Brainstorm at least one mathematical approach you could take towards having students flesh this concept out

Summary

Challenges for Educators

- Perceptions Re: *mathematics*
- Cultural differences across different areas of study
- Integrating math content into life sciences courses & vice versa

Challenges for Students

- Expectations coming into/out of math courses ⇒ critical thinking skills

Strategies for Educators

1. Incorporate more biological content into math courses
   (via a broad range of contextual and realizable applications)

2. Develop effective mathematics *refresher* sessions for life sciences faculty; conversely, biology refreshers for math faculty

3. Consider how mathematical content can be integrated into subsequent biology courses

4. Advising (e.g., helping students identify their strengths/interests)

Fini

Questions? Comments? Ideas?

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Slides/Questionnaire downloadable at:

http://math.arizona.edu/~cbergevin/

Financial Support: National Science Foundation, Howard Hughes Medical Institute

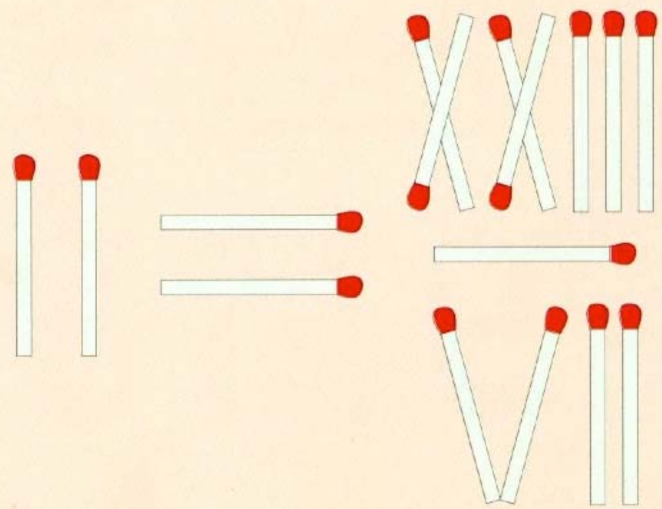
Two people were discussing the matchstick puzzle shown below. The idea was to move only one match to make the statement correct.

"I reckon it's impossible to make it exactly correct," claimed Kevin, "unless you cheat by turning the equals sign into a 'not equal to' sign."

"Maybe," said Mandy, "but I can make it approximately correct." She made her move and the result was only a few percent from being exactly correct.

"Aha!" exclaimed Kevin. "I can move the same match as you've just moved to make the statement even closer to being exactly correct."

Can you work out what the two solutions were?



Source: 'Mind Benders: Adventures in Lateral Thinking'
by David J. Bodycombe

Question:

What is mathematics?

`For scholars and layman alike it is not philosophy but active experience in mathematics itself that alone can answer the question: What is mathematics?'

- Courant & Robbins (1941)

Answer #1: *Educators View*

Differences in perception Re: *mathematics*

math faculty ⟷ life sciences faculty

"the science that draws necessary conclusions"
- B. Peirce

'quantitative literacy'
(i.e., a tool or language)

Need to be '*mindful of cultural differences across departments*'

- HHMI external review committee

<u>Answer #1:</u> Caveat

Differences in perception Re: *biology*

life sciences faculty ⟷ math faculty

an evolutionary science
(e.g., first ask *how*, then *why*)

"science fiction"

Answer #2: *Students View*

Expectations coming into a mathematics course:

learning/doing math	vs ⟷	developing *analytical* & *quantitative* reasoning skills
'taught to the test'		critical thinking

Examples To Emphasize Critical Thinking [How vs. Why]

$$\int_{-1}^{1} \sqrt{1 - \delta^2}\, d\delta$$

Means to solve this integral?

1. Table of integrals
2. Numerically (e.g., estimate the area)
3. Use a computer (or even a phone!)
4. Taylor series expansion
5. Trigonometric substitution and integration by parts

Examples To Emphasize Critical Thinking [How vs. Why]

- 'Word problems'

  (and associated jargon)

- *Modeling*

- Issue of ***translation***

Ex. (from an ODE text)

Suppose a population has a constant per unit death rate (d>0) and a per unit birth rate that is proportional to the population concentration x (with a constant of proportionality denoted by a>0). Using the balance law, write a differential equation for the population concentration $x(t)$.

$$\frac{dx}{dt} = x(ax - d)$$

Basic Idea

Examples

Short Refresher: *Types* of Mathematics

Differential equations

Deals with how something changes w/ respect to something(s) else

Law of Mass Action, Michaelis-Menten kinetics

Discrete Mathematics

Deals w/ discrete #s of objects

Deals with molecular events

Calculus

'Study of change'

Everywhere!

Linear Algebra

Linear systems*

Everywhere!

Statistics & Probability

Handles what we can't know

Data analysis

Stochastic Dynamics

Random, noisy processes

Noise, irregularity

Vector Calculus

Multi-Variable functions

Micro-fluidics

Fourier Analysis

Deals with oscillatory behavior

Signal processing, transforms